The Effects of Aggregation and Elasticities on Optimal Tariffs in a Nash Tariff-Setting Game*

Robert Waschik
La Trobe University

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Abstract

It is typical to incorporate trade into open-economy Computable General Equilibrium (CGE) models by modelling imports from different countries as differentiated products using the so-called Armington assumption. When these Armington functions are modelled as C.E.S. functions, import demand elasticities can be approximated by the substitution elasticity of the C.E.S. (Armington) function in which regional imports enter as an input. This method of calibrating CGE models to estimated import demand elasticities sourced from a relevant empirical econometrics literature is particularly important to results of counterfactual experiments of trade policy changes in many CGE modelling exercises. And of course, such import demand elasticities are endogenous in any general equilibrium model. The objective of this paper is to investigate the implications of using this approximation method to calibrate open-economy CGE models to import demand elasticities, to determine if and when the fact that the substitution elasticity only approximately reflects the import demand elasticity results in non-trivial calibration errors. We also describe how these import demand elasticities respond endogenously to a particular trade policy experiment, where a (non-small) open economy charges an optimal tariff on imports, since it is well-known how the optimal tariff depends upon such trade elasticities.

*This is work in progress, and should not be quoted without prior approval of the author. Correspondence should be directed to the author at the Department of Economics and Finance, La Trobe University, Melbourne, Australia, 3086, or via email to `r.waschik@latrobe.edu.au`.

1 Introduction

It is a well-known result in trade theory that a nation's optimal import tariff is equal to the reciprocal of its trading partner's import demand elasticity minus 1 (see Johnson (1965:58), among many others). A considerable literature presumes that trading nations charge optimal tariffs on each other's imports. For example, theoretical and applied models of Preferential Trade Agreements (PTAs) generally assume that PTA members charge optimal tariffs on imports from non-member countries. This paper examines the usefulness of the simple inverse elasticity formula in expressing a nation's optimal tariff on imports in Computable General Equilibrium (CGE) models.

It is common to model trade in CGE models by adopting the so-called Armington assumption, whereby the same good produced by a different country is treated as a differentiated product. Such models are often implemented by using a series of nested C.E.S. functions to represent consumer behaviour. For example, a representative consumer's preferences over consumption of goods $i = 1, \dots n$ is represented by a C.E.S. utility function. Any good i is a C.E.S. aggregate of domestic i and imported i, where the imported i itself can be a C.E.S. aggregate of good i imported from a country's trading partners. Such CGE models are calibrated to econometrically estimated import demand elasticities using the substitution elasticities in these C.E.S. functions: The import demand elasticity for good i is approximately equal to the C.E.S. substitution elasticity in the lowest-level nest below good i. But using this approximation of a country's import demand elasticity in the optimal tariff formula provides a poor approximation of a nation's optimal tariff.

The objective of this paper is to clearly present a number of reasons for this result. First, the C.E.S. substitution elasticity will only be approximately equal to a country's import demand elasticity. Second, while the C.E.S. substitution elasticity is exogenously specified in CGE models, import demand elasticities will depend upon trade shares and prices which are endogenous. If a country is not applying an optimal tariff in the initial equilibrium in the CGE model, then the import demand elasticity will vary endogenously in moving to the optimal tariff equilibrium. Finally, the import demand elasticity to which CGE models are typically benchmarked is an uncompensated own-price import demand elasticity, while the import demand elasticity cited in the optimal tariff formula is a "general equilibrium" elasticity, where the percentage change in imports is due to a percentage change in a country's terms-of-trade after "all the repercussions of general equilibrium adjustment have been worked out" (Johnson (1965:60-1), footnote 6).

The plan of the paper is as follows. In Section 2, we describe a series of typical general equilibrium trade models and solve for the relevant trade elasticities. We begin in Section 2.1 with the standard two-good, two-factor, two-country Heckscher-Ohlin trade model. In Section 2.2 we implement the Armington assumption that the same good produced in different countries is viewed by consumers as a differentiated product, and show how the relevant trade elasticities change. We then review the optimal tariff problem in Section 3, deriving the well-known inverse elasticity formula, and also deriving an expression for the optimal tariff as a function of compensated trade elasticities. We construct an arbitrary general equilibrium data set and use it to solve the optimal tariff problem, in the Heckscher-Ohlin model in Section 3.1 and in the Armington model in Section 3.2. This allows us to relate the actual trade elasticities in the optimal tariff equilibrium to those in the initial equilibrium, as well as to the C.E.S. substitution elasticity which is used to calibrate the CGE model. The implications of these results for large-scale CGE models are discussed in Section 3.3. Concluding comments are offered in Section 4.

2 General Equilibrium Model

We begin with a standard two-good, two-factor, two-country Heckscher-Ohlin general equilibrium trade model. Both countries have the same production technology: Trade is motivated by international differences in relative endowments of factors of production. Both goods are traded but factors of production are immobile between countries. Factor endowments are exogenous. In each economy, a representative consumer earns income from ownership of factors of production. We implement this model using a Computable General Equilibrium (CGE) model where consumers have preferences represented by a C.E.S. utility function. In Section 2.1 we describe the typical model where countries produce homogeneous goods. We present the model where consumers regard the same good produced by different countries as differentiated products in Section 2.2. In either case, we assume throughout that input and output markets are perfectly competitive, production technology displays constant returns to scale and unit production functions are represented by Cobb-Douglas functions:

$$1 \;=\; A_i K_i^{\alpha_i} L_i^{\beta_i} \qquad \alpha_i + \beta_i = 1 \qquad i \;=\; x, y.$$

Cost minimization implies that the corresponding input demand functions can be written as:

$$K_i \;=\; \left(\frac{\alpha_i}{\beta_i}\frac{w}{r}\right)^{\beta_i} / A_i$$

$$L_i = \left(\frac{\beta_i}{\alpha_i}\frac{r}{w}\right)^{\alpha_i} / A_i \qquad i = x, y.$$

We can solve for output prices as a function of input prices by substituting these input demand functions into each firm's zero-profit condition:

$$p_i = w^{\beta_i} r^{\alpha_i} \alpha_i^{-\alpha_i} \beta_i^{-\beta_i} / A_i \qquad i = x, y.$$

Now write the input demand functions as a function of output prices by inverting the zero-profit conditions to solve for the wage/rent ratio, and substituting in the input demand functions:

$$\begin{aligned}
K_x &= \left(\frac{p_x}{p_y}\right)^{\frac{\beta_x}{k}} A_x^{\frac{\beta_y}{k}} A_y^{\frac{-\beta_x}{k}} \left(\frac{\alpha_x}{\alpha_y}\right)^{\frac{\alpha_y\beta_x}{k}} \left(\frac{\beta_x}{\beta_y}\right)^{\frac{\beta_x\beta_y}{k}} \\
L_x &= \left(\frac{p_x}{p_y}\right)^{\frac{-\alpha_x}{k}} A_x^{\frac{-\alpha_y}{k}} A_y^{\frac{\alpha_x}{k}} \left(\frac{\alpha_x}{\alpha_y}\right)^{\frac{-\alpha_x\alpha_y}{k}} \left(\frac{\beta_x}{\beta_y}\right)^{\frac{-\alpha_x\beta_y}{k}} \\
K_y &= \left(\frac{p_x}{p_y}\right)^{\frac{\beta_y}{k}} A_x^{\frac{\beta_y}{k}} A_y^{\frac{-\beta_x}{k}} \left(\frac{\alpha_x}{\alpha_y}\right)^{\frac{\alpha_x\beta_y}{k}} \left(\frac{\beta_x}{\beta_y}\right)^{\frac{\beta_x\beta_y}{k}} \\
L_y &= \left(\frac{p_x}{p_y}\right)^{\frac{-\alpha_y}{k}} A_x^{\frac{-\alpha_y}{k}} A_y^{\frac{\alpha_x}{k}} \left(\frac{\alpha_x}{\alpha_y}\right)^{\frac{-\alpha_x\alpha_y}{k}} \left(\frac{\beta_x}{\beta_y}\right)^{\frac{-\alpha_y\beta_x}{k}}
\end{aligned}$$

where $k = \alpha_y\beta_x - \alpha_x\beta_y$ reflects relative factor intensity in production. To solve for output supply functions, we invert the full employment conditions:

$$\begin{pmatrix} K_x & K_y \\ L_x & L_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \bar{K} \\ \bar{L} \end{pmatrix}$$

and substitute the input demand functions to solve for output supplies as a function of endowments and output prices:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \left(\frac{-\alpha_x\beta_y}{k}\right)\left(\frac{\alpha_y\beta_x}{\alpha_x\beta_y}\right)^{\frac{\alpha_y\beta_x}{k}} \begin{pmatrix} \left(\frac{\beta_x}{\beta_y}\frac{p_x}{p_y}\right)^{\frac{-\beta_x}{k}} A_x^{\frac{-\beta_y}{k}} A_y^{\frac{\beta_x}{k}} \cdot \bar{K} - \left(\frac{\alpha_x}{\alpha_y}\frac{p_x}{p_y}\right)^{\frac{\alpha_x}{k}} A_x^{\frac{\alpha_y}{k}} A_y^{\frac{-\alpha_x}{k}} \cdot \bar{L} \\ -\left(\frac{\beta_x}{\beta_y}\frac{p_x}{p_y}\right)^{\frac{-\beta_y}{k}} A_x^{\frac{-\beta_y}{k}} A_y^{\frac{\beta_x}{k}} \cdot \bar{K} + \left(\frac{\alpha_x}{\alpha_y}\frac{p_x}{p_y}\right)^{\frac{\alpha_y}{k}} A_x^{\frac{\alpha_y}{k}} A_y^{\frac{-\alpha_x}{k}} \cdot \bar{L} \end{pmatrix}$$

These output supply functions can be used to solve for the output supply elasticities which will be required to solve for trade elasticities.

2.1 Demand Elasticities When Traded Goods are Homogeneous

If we represent preferences in the simple two-good, two-country Heckscher-Ohlin model with a C.E.S. utility function, the consumer's problem can be written as:

$$\max_{z_i} U(z_i) = \left(\sum_{i=1}^{n} \alpha_i z_i^{-\rho}\right)^{-1/\rho} \qquad \text{subject to: } I \geq p'z$$

where z_i is demand for commodity i and income $I = w\bar{L} + r\bar{K}$. Solving this problem yields (uncompensated) output demand functions:

$$z_j = \frac{\alpha_j^\sigma I}{p_j^\sigma \sum_i \alpha_i^\sigma p_i^{1-\sigma}},$$

where the elasticity of substitution $\sigma = \frac{1}{1+\rho}$. It is a straightforward matter to solve for the (uncompensated) demand elasticities:[1]

$$\frac{\partial z_j}{\partial p_j}\frac{p_j}{z_j} = \epsilon_j = -\sigma - (1-\sigma)\frac{\alpha_j^\sigma p_j^{1-\sigma}}{\sum_i \alpha_i^\sigma p_i^{1-\sigma}} = -\sigma - (1-\sigma)S_j, \tag{1}$$

where $S_j = p_j z_j / I$ is the share of income spent on good j. If this share is small, then the demand elasticity for good j can be approximated by the C.E.S. substitution elasticity σ.

We will ultimately be interested in compensated trade elasticities, for which we will need compensated demand functions. To solve for the income effect, note that z_j is linear in I, $(\partial z_j / \partial I) \cdot I = z_j$, so:

$$\frac{\partial z_j}{\partial I}\frac{\partial I}{\partial p_j}\frac{p_j}{z_j} = \frac{z_j p_j}{I} = \frac{\alpha_j^\sigma p_j^{1-\sigma}}{\sum_i \alpha_i^\sigma p_i^{1-\sigma}} = S_j,$$

This allows us to write the compensated demand elasticity $\tilde{\epsilon}_j$ as a function of expenditure shares:

$$\tilde{\epsilon}_j = -\sigma \cdot (1 - S_j)$$

These compensated demand elasticities and the supply elasticities derived earlier can be used to derive expressions for compensated trade elasticities which will be used to express a country's optimal tariff.

It is also common to express optimal tariffs as a function of a country's offer curve elasticity. We follow the derivation in Zhang (2006) to solve for the offer curve elasticity in a two-good two-country model with fixed production (exchange economies). Suppose the Home country exports good i, imports good j, and applies an *ad valorem* tariff t_j on imports of good j. Letting $\bar{q}_i$ ($\bar{q}_j$) denote the (fixed) production of good i (j), and using a * to denote world prices, $p_i = p_i^*$ and $p_j = p_j^*(1+t_j)$, we substitute the income-equals-expenditure constraint:

$$\begin{aligned} p_i^*\bar{q}_i + p_j^*\bar{q}_j + p_j^* t_j z_j &= p_i^* z_i + p_j^*(1+t_j) z_j \\ \frac{p_i^*}{p_j^*(1+t_j)} &= \frac{(z_j - \bar{q}_j)}{(\bar{q}_i - z_i)\cdot(1+t_j)} \end{aligned}$$

[1]See Mansur and Whalley (1984:106-7), for example.

into the expression for the ratio of any two demand functions:

$$\begin{aligned} z_j/z_i &= (\alpha_j/\alpha_i)^\sigma \left(\frac{p_i^*}{p_j^*(1+t_j)}\right)^\sigma \\ z_j/z_i &= (\alpha_j/\alpha_i)^\sigma \left(\frac{(z_j-\bar{q}_j)}{(\bar{q}_i-z_i)\cdot(1+t_j)}\right)^\sigma \end{aligned}$$

which is the same as equation (14) on p.7 of Zhang (2006). We can find the elasticity of the offer curve ϵ_{oc} by rewriting this expression as:

$$0 = (\alpha_j/\alpha_i)^\sigma \left(\frac{(z_j-\bar{q}_j)}{(\bar{q}_i-z_i)\cdot(1+t_j)}\right)^\sigma \frac{z_i}{z_j} - 1$$

and applying the implicit function theorem to get:

$$\frac{\partial(z_j-\bar{q}_j)/(z_j-\bar{q}_j)}{\partial(\bar{q}_i-z_i)/(\bar{q}_i-z_i)} = \epsilon_{oc} = \frac{\sigma+(\bar{q}_i-z_i)/z_i}{\sigma+(\bar{q}_j-z_j)/z_j} \quad (2)$$

2.2 Demand Elasticities When Traded Goods are Differentiated Products

Suppose instead that the representative consumer in each country regards the same good produced in different countries as imperfect substitutes, so countries produce differentiated products. This model, described in Armington (1969), is illustrated in Figure 1, where σ is the upper level C.E.S. substitution elasticity between aggregate goods x and y in consumption, and σ_i is the lower level C.E.S. substitution elasticity between the same good produced in different countries or regions.

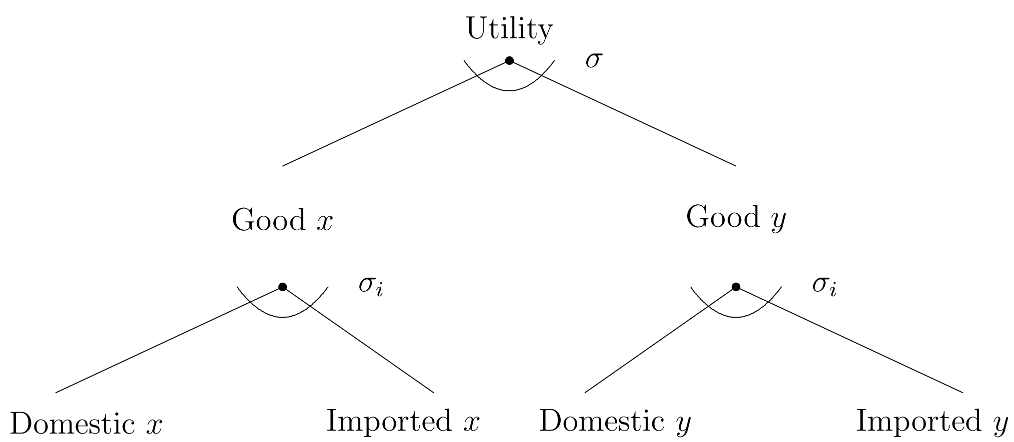

Figure 1: Structure of Consumption in a Simple Armington Model

The upper level utility maximization problem is the same as that in Section 2.1. Following Armington (1969), the lower level maximization problem represents the consumer's choice between domestic and imported varieties in good z_i, given by:

$$z_i = \left[\alpha_{id} z_{id}^{-\rho_i} + \alpha_{im} z_{im}^{-\rho_i}\right]^{-1/\rho_i} \qquad \text{subject to:} \quad I_i = p_i z_i$$
$$z_{ik} = \frac{\alpha_{ik}^{\sigma_i} I_i}{p_{ik}^{\sigma_i}\left[\alpha_{id}^{\sigma_i} p_{id}^{1-\sigma_i} + \alpha_{im}^{\sigma_i} p_{im}^{1-\sigma_i}\right]} \qquad k = d, m,$$

where $\sigma_i = \frac{1}{1+\rho_i}$ is the elasticity of substitution between domestic and imported varieties of good i, and the price index for the i'th aggregate commodity is given by $p_i = \left[\alpha_{id}^{\sigma_i} p_{id}^{1-\sigma_i} + \alpha_{im}^{\sigma_i} p_{im}^{1-\sigma_i}\right]^{\frac{1}{1-\sigma_i}}$. We can solve for the (uncompensated) own-price elasticity of demand for the imported good z_{im} to get:

$$\frac{\partial z_{im}}{\partial p_{im}}\frac{p_{im}}{z_{im}} = -\sigma_i - S_{im}(1-\sigma_i) + S_{im}(1+\epsilon_i)$$
$$\frac{\partial z_{im}}{\partial p_{im}}\frac{p_{im}}{z_{im}} = -\sigma_i + S_{im}(\sigma_i + \epsilon_i), \tag{3}$$

where $S_{im} = \frac{p_{im} z_{im}}{p_i z_i}$ is spending on good z_{im} out of total spending on aggregate i, $\epsilon_i = -\sigma - (1-\sigma)S_i$ is the demand elasticity for aggregate good i from Section 2.1, and $S_i = p_i z_i / M$ is spending on aggregate good i out of total income.[2] Likewise the cross-elasticity of demand for good z_{im} with respect to the price of the domestic variety of the good in the i'th nest, p_{id}, is:

$$\frac{\partial z_{im}}{\partial p_{id}}\frac{p_{id}}{z_{im}} = -S_{id}(1-\sigma_i) + S_{id}(1+\epsilon_i) = S_{id}(\sigma_i + \epsilon_i),$$

and the cross-elasticity with respect to another aggregate price p_k is:

$$\frac{\partial z_{im}}{\partial p_k}\frac{p_k}{z_{im}} = -S_k(1-\sigma) \qquad \forall\ k \neq i.$$

As was the case for the upper-level demand functions, the income effects for the lower-level demand functions are all equal to their respective expenditure shares:

$$\frac{\partial z_{im}}{\partial I}\frac{\partial I}{\partial p_{im}}\frac{p_{im}}{z_{im}} = \frac{z_{im} p_{im}}{I}$$
$$\frac{\partial z_{im}}{\partial I}\frac{\partial I}{\partial p_{id}}\frac{p_{id}}{z_{im}} = \frac{z_{id} p_{id}}{I}$$
$$\frac{\partial z_{im}}{\partial I}\frac{\partial I}{\partial p_k}\frac{p_k}{z_{im}} = \frac{z_k p_k}{I} \qquad \forall\ k \neq i.$$

[2]This elasticity and the following two cross-price elasticities are the same as the corresponding bracketed coefficients in equation (26) of Armington (1969:175), except that the terms on the direct price elasticity of demand ϵ_i in Armington are of opposite sign. This stems from the definition of the direct price elasticity of demand for X_i in equation (24) of Armington (1969:174), which must be defined in absolute value. Also note that Mansur and Whalley (1984:107) approximate this lower-level own-price elasticity as: $\frac{\partial z_i}{\partial p_i}\frac{p_i}{z_i} \approx -\sigma_k - \alpha_{jk}(1-\sigma_k) - \alpha_{jk}(1-\sigma)[-\sigma - \alpha_k(1-\sigma)]$, where commodity i is the j'th commodity in the k'th nest.

3 The Optimal Tariff Problem

We begin with a brief review of the optimal tariff problem, as presented in Johnson (1965:56-61) and Graaff (1949:52-4). In a simple two-good two-country trade model, the Home country's optimal tariff t^{opt} will shift its offer curve to intersect that of its trading partner where the Home country's trade indifference curve is tangent to its trading partner's offer curve. This is illustrated in Figure 2 by point P, where t^{opt} satisfies $\frac{p_x}{p_y} = \frac{p_x^*}{p_y^* \cdot (1+t^{opt})}$. Since the world (domestic) price of x is given by the slope of the ray OP (PR), $t^{opt} = (OQ/PQ)/(QR/PQ) - 1 = OQ/QR - 1$. By definition, the elasticity of the Foreign country's offer curve is $\eta_{oc}^F = \frac{dx/x}{dy/y} = \frac{y}{x}\frac{dx}{dy}$. Evaluated at P, $\eta_{oc}^F = (OQ/PQ) \cdot (PQ/QR) = OQ/QR$, so when the Home country is charging its optimal tariff, it must be the case that $t^{opt} = \eta_{oc}^F - 1$.[3]

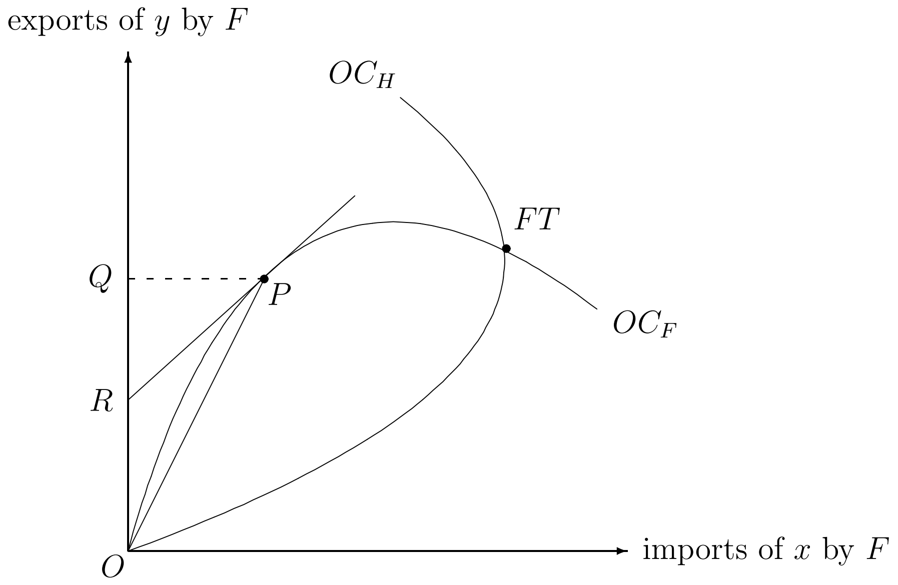

Figure 2: Home's Optimal Tariff on Imports of y

To derive the familiar inverse-elasticity rule, define the Foreign country's import demand elasticity $\eta_x^F = \frac{dx/x}{d(p_x^*/p_y^*)/(p_x^*/p_y^*)}$. Since Foreign's balance of payments constraint implies that $p_x^*/p_y^* = y/x$, its import demand elasticity can be written in terms of the elasticity of its offer curve: $\eta_x^F = \frac{\eta_{oc}^F}{\eta_{oc}^F - 1}$. This allows the optimal tariff to be written as:

$$t^{opt} = \frac{1}{\eta_x^F - 1} :$$

[3] As pointed out in Johnson (1965) and others, when the elasticity of the Foreign country's offer curve η_{oc}^F is constant, it is a straightforward matter to solve for the Home country's optimal tariff. But in general, η_{oc}^F will be endogenous.

"The optimum welfare tariff rate is equal to the reciprocal of one less than the foreign elasticity of demand for exports as a function of the barter terms of trade", Johnson (1965:59).

At this point it is important to note that Foreign's import demand elasticity η_x^F is written as a function of the terms of trade. Of course, the elasticities derived in Section 2 are derived as a function of nominal prices. Johnson (1965:59-61) addresses this issue by defining Foreign's elasticity of demand for imports of x, $\epsilon_x^F = -\frac{p_x}{x}\frac{dx}{dp_x}$ and Foreign's elasticity of supply of exports of y, $\epsilon_y^F = -\frac{p_y}{y}\frac{dy}{dp_y}$, and solving for the optimal tariff formula by differentiating Foreign's balance of payments constraint to get:

$$t^{opt} = \frac{\frac{1}{\epsilon_y} + \frac{1}{\epsilon_x}}{1 - \frac{1}{\epsilon_x}}$$

This is equation (5) in Johnson (1965:60) and equation (3) in Graaff (1949:54). In fact, even this formula is misleading. Johnson (1965) (in footnote 6 on pp.60-1) and Graaff (1949:54) both note that this expression needs to be interpreted with care, since in deriving this formula, it is assumed that "all cross elasticities vanish identically" and that all goods are independent in production (i.e. all productive factors are completely specific). Even in an exchange economy, the use of C.E.S. functions to model consumer preferences precludes the ability to apply the assumption that "all cross price elasticities vanish identically".

To solve for the optimal tariff formula as a function of partial elasticities in an environment where cross elasticities are non-zero, begin with the Balance of Payments constraint: $p_m m(p_e, p_m) = p_e e(p_e, p_m) + t_m p_m^* m(p_e, p_m)$, where exports are defined as $e(p_e, p_m, v, \mu) = y^e(p_e, p_m, v) - z^e(p_e, p_m, \mu)$, imports as $m(p_e, p_m, v, \mu) = z^m(p_e, p_m, \mu) - y^m(p_e, p_m, v)$, and y^i and z^i are the output supply and compensated demand functions, respectively. Take a derivative with respect to the import tariff t_m, which if chosen optimally, will impy that $\partial\mu/\partial t_m = 0$ (factor endowments v are assumed to remain constant):[4]

$$\frac{1+t_m}{t_m}\left\{[1+\tilde{\epsilon}_{mm} - \tilde{\epsilon}_{em}]\frac{\partial p_m^*/p_m^*}{\partial t_m/t_m} - [1+\tilde{\epsilon}_{ee} - \tilde{\epsilon}_{me}]\frac{\partial p_e^*/p_e^*}{\partial t_m/t_m}\right\} = \tilde{\epsilon}_{em} - \tilde{\epsilon}_{mm}$$

We can choose one price as numeraire, so suppose $\frac{\partial p_e^*/p_e^*}{\partial t_m/t_m} = 0$. This allows us to write the optimal tariff as:

$$t_m = \frac{[1+\tilde{\epsilon}_{mm} - \tilde{\epsilon}_{em}]\frac{\partial p_m^*/p_m^*}{\partial t_m/t_m}}{\tilde{\epsilon}_{em} - \tilde{\epsilon}_{mm} - [1+\tilde{\epsilon}_{mm} - \tilde{\epsilon}_{em}]\frac{\partial p_m^*/p_m^*}{\partial t_m/t_m}} \qquad (4)$$

[4]See the Appendix for a detailed derivation of this expression.

We are interested in highlighting two important (though rather obvious) points which need to be kept in mind when solving for optimal tariffs in CGE models:

- partial versus general equilibrium trade elasticities necessitate the use of different optimal tariff formulae, and

- elasticities are typically endogenous.

The latter point is particularly important to keep in mind since CGE trade models typically use C.E.S. functions to represent demand behaviour. Such models are commonly calibrated to econometrically estimated demand elasticities using data on the value of consumption, production, and trade, and equations like (1) and (3) by exogenous specification of the C.E.S. substitution elasticities σ and σ_i.[5] Even though these substitution elasticities are exogenous, the demand elasticities depend upon value shares which can vary considerably, particularly when optimal tariffs are a fair distance from those in an initial equilibrium. Approximating demand elasticities using C.E.S. substitution elasticities like those in equations (1) and (3), and then using those approximations to generate back-of-the-envelope estimates of optimal tariffs, can yield very poor results.

3.1 Solving the Optimal Tariff Problem in the Heckscher-Ohlin Model

To illustrate, we begin with a simple CGE model of the two good, two factor, two country Heckscher-Ohlin trade model described in section 2.1. We construct an arbitrary general equilibrium data set with all of the characteristics of the simple Heckscher-Ohlin model. This data set is represented in a Social Accounting Matrix (SAM) in Table 1 which summarizes all quantities of input usage, production, consumption, and trade for an initial general equilibrium where:

- demand equals supply for all goods and factors,
- profits equal zero in each industry,
- income equals expenditure for each (representative) consumer, and
- trade is balanced.

Note that both countries have the same production technology, represented with Cobb-Douglas production functions. Each country produces i (j) using the same ratio of labour to capital.[6] The representative consumer in each country has the same preferences, modelled using a C.E.S. utility function where the substitution elasticity is set

[5] See Dawkins *et al* for a description of calibration methods in CGE models.

[6] In the initial equilibrium all input and output prices are equal to unity, so the wage/rent ratio in each country in the initial equilibrium is the same.

Home	good i	good j	consump	export	import	demand
good i			100	50	0	150
good j			100	0	50	50
labour	100	20				
capital	50	30				
supply	150	50				

Foreign	good i	good j	consump	export	import	demand
good i			100	0	50	50
good j			100	50	0	150
labour	33.3	60				
capital	16.7	90				
supply	50	150				

Table 1: Social Accounting Matrix for two-good two-factor Heckscher-Ohlin Economy

arbitrarily at $\sigma = 4$. Home is relatively well endowed with labour, and exports good i in the initial equilibrium, the good which is produced using intensively Home's relatively abundant factor.

To consider the simplest model, suppose that both labour and capital are specific in production and immobile between production sectors, so that the level of production is fixed. If Home charges an optimal tariff on imports of good j, it would set $t = 0.189$ or 18.9%. We use the equations in Section 2.1 to evaluate the relevant elasticities at both the initial equilibrium in Table 1 and the equilibrium where Home charges an optimal tariff. These elasticities are presented in Table 2.

Clearly the output supply elasticities are equal to zero, since production is fixed. Given the expenditure shares in the initial equilibrium and the exogenously specified substitution elasticity $\sigma = 4$, the uncompensated demand elasticities are -2.5 for each good at the initial equilibrium. Of course, σ is typically chosen to calibrate the model to a particular demand elasticity. For example, if we had information that the uncompensated elasticity of demand for good i was equal to $\epsilon_i = -2.5$, we would set $\sigma = 4$. It should be clear that using the C.E.S. function precludes the ability to calibrate to specific elasticities for all goods.[7] Once we choose $\sigma = 4$ to calibrate the model to $\epsilon_i = -2.5$, this implies a value for ϵ_j. It would be a fortunate coincidence if econometric estimates of ϵ_j suggested we calibrate our model so that $\epsilon_j = -2.5$.

We can use equation (2) to solve for the offer curve elasticity at the initial equilibrium and the optimal tariff equilibrium, and confirm that the Home country's optimal tariff

[7]See Perroni and Rutherford (1998) on using flexible functional forms to calibrate CGE models to a matrix of demand elasticities.

	Evaluated at initial equilibrium				Evaluated at optimal import tariff			
	supply elasticities				supply elasticities			
	Home		Foreign		Home		Foreign	
	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	uncompensated demand elasticities				uncompensated demand elasticities			
	Home		Foreign		Home		Foreign	
	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2
1	-2.5000	1.5000	-2.5000	1.5000	-2.3093	1.3093	-2.6964	1.6964
2	1.5000	-2.5000	1.5000	-2.5000	1.6907	-2.6907	1.3036	-2.3036
	compensated demand elasticities				compensated demand elasticities			
	Home		Foreign		Home		Foreign	
	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2
1	-2.0000	2.0000	-2.0000	2.0000	-1.7457	1.7457	-2.2618	2.2618
2	2.0000	-2.0000	2.0000	-2.0000	2.2543	-2.2543	1.7382	-1.7382
	uncompensated trade elasticities				uncompensated trade elasticities			
	Home		Foreign		Home		Foreign	
	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2
1	5.0000	-3.0000	-5.0000	3.0000	8.7113	-4.9389	-6.9857	4.3949
2	3.0000	-5.0000	-3.0000	5.0000	4.1543	-6.6114	-4.3949	7.7662
	compensated trade elasticities				compensated trade elasticities			
	Home		Foreign		Home		Foreign	
	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2
1	4.0000	-4.0000	-4.0000	4.0000	6.5853	-6.5853	-5.8599	5.8599
2	4.0000	-4.0000	-4.0000	4.0000	5.5390	-5.5390	-5.8599	5.8599
	offer curve elasticity				offer curve elasticity			
	Home		Foreign		Home		Foreign	
	1.28571429		1.28571429		1.18705066		1.18887875	
	import demand elasticity				import demand elasticity			
	Home		Foreign		Home		Foreign	
	4.50000000		4.50000000		6.34614533		6.29440175	

Table 2: Elasticities in Heckscher-Ohlin Exchange Economy

is equal to the elasticity of the Foreign Country's offer curve minus one. Likewise we can solve for each country's import demand elasticity with respect to the terms of trade, and confirm the inverse elasticity rule: $t = 1/(\eta_i^F - 1)$.

Of course, the import demand elasticity with respect to the terms of trade is not equal to the (compensated) import demand elasticity. To express Home's optimal tariff as a function of the compensated trade elasticities in Table 2, we need to substitute Home's compensated trade elasticities into equation (4).[8] Note as well that the elasticities are clearly not constant: Import demand is more elastic in the optimal tariff equilibrium.

To consider the more general version of the Heckscher-Ohlin model where production is variable, we presume that labour and capital are homogeneous and perfectly mobile between industries i and j. We consider the same initial general equilibrium summarized in Table 1. Elasticities for this model are given in Table 3. In the initial equilibrium, the demand elasticities are necessarily the same as those reported in the initial equilibrium in Table 2, but since the supply elasticities are now non-zero, the trade elasticities are also different.

With endogenous production, the optimal tariff that Home charges on imports of good 2 from Foreign is $t = 0.036$ or 3.6%. As is clear from the elasticities reported in Table 3, the assumption of Cobb-Douglas production technology results in large output supply elasticities, which imply that trade elasticities are considerably larger than those in the exchange-economy case. These much larger trade elasticities result in a much smaller optimal tariff.

3.2 Solving the Optimal Tariff Problem in the Armington Model

We now adapt the general equilibrium model we have been using thus far to accommodate the assumption that the same good produced in different countries is viewed by consumers as a differentiated product. To begin with, we will consider the simplest such general equilibrium model, where each country produces only one good, so supply elasticities must all be equal to zero. The initial equilibrium is summarized in the Social Accounting Matrix in Table 4.[9]

As before, preferences are summarized by a C.E.S. utility function where the substitution elasticity is set equal to $\sigma = 4$. In this example, the Home country is larger than the Foreign country, and even in the initial equilibrium, demand elasticities for

[8] To apply equation (4), we also need to substitute for the effect of a change in Home's tariff on the world price of the imported good $\frac{\partial p_m^*/p_m^*}{\partial t_m/t_m}$.

[9] This model is equivalent to that in de Melo and Robinson (1989), except they also accommodate a transformation elasticity between domestic and export goods which is less than infinity.

	Evaluated at initial equilibrium				Evaluated at optimal import tariff			
	supply elasticities				supply elasticities			
	Home		Foreign		Home		Foreign	
	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2
1	4.2500	-4.2500	13.2500	-13.2500	4.7784	-4.7784	9.9978	-9.9978
2	-12.7500	12.7500	-4.4167	4.4167	-9.7538	9.7538	-4.9096	4.9096
	uncompensated demand elasticities				uncompensated demand elasticities			
	Home		Foreign		Home		Foreign	
	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2
1	-2.5000	1.5000	-2.5000	1.5000	-2.4479	1.4479	-2.5509	1.5509
2	1.5000	-2.5000	1.5000	-2.5000	1.5521	-2.5521	1.4491	-2.4491
	compensated demand elasticities				compensated demand elasticities			
	Home		Foreign		Home		Foreign	
	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2
1	-2.0000	2.0000	-2.0000	2.0000	-1.9305	1.9305	-2.0678	2.0678
2	2.0000	-2.0000	2.0000	-2.0000	2.0695	-2.0695	1.9322	-1.9322
	uncompensated trade elasticities				uncompensated trade elasticities			
	Home		Foreign		Home		Foreign	
	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2
1	17.7500	-15.7500	-18.2500	16.2500	29.6796	-26.2337	-29.3099	26.1775
2	15.7500	-17.7500	-16.2500	18.2500	25.3103	-28.4117	-26.4392	29.8251
	compensated trade elasticities				compensated trade elasticities			
	Home		Foreign		Home		Foreign	
	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2
1	16.7500	-16.7500	-17.2500	17.2500	27.8968	-27.8968	-27.7968	27.7968
2	16.7500	-16.7500	-17.2500	17.2500	26.9149	-26.9149	-28.0747	28.0747

Table 3: Elasticities in Heckscher-Ohlin Production Economy

domestic and imported goods differ between Home and Foreign. All demand elasticities are presented in Table 5.[10]

When Home charges an optimal tariff on imports from Foreign, it sets its tariff equal to $t = 0.495$ or 49.5%. We can use the elasticities evaluated at the optimal tariff equilibrium in Table 5 to verify the optimal tariff formulae derived at the beginning of Section 3. It is worth noting that in this highly aggregated model, even when the supply elasticities are equal to zero, the C.E.S. substitution elasticity $\sigma = 4$ is a poor approximation of either country's import demand elasticity, neither in the initial equilibrium nor when Home charges an optimal tariff.

We now move on to the two-good, two-factor, two-country Armington model, represented by the Social Accounting Matrix in Table 6. We adopt the same SAM as was

[10] Note that Home produces good 1, Foreign produces good 2.

Home	good i	consump	export	import	demand
good i		100	50	50	150
labour	100				
capital	50				
supply	150				

Foreign	good i	consump	export	import	demand
good i		50	50	50	100
labour	66.7				
capital	33.3				
supply	100				

Table 4: Social Accounting Matrix for one-good two-factor Armington Economy

used in the previous Section, so output supply and input demand in Table 6 are the same as in Table 1. But because consumers regard the same good produced in Home and Foreign as differentiated products, both Home and Foreign import and export both goods, so the structure of trade in Table 6 is different from that in Table 1. This implies that while the representative consumer in Home consumes 100 units of good i in both Table 1 and Table 6, in the Heckscher-Ohlin version of this model, this represented 100 units of consumtion of good i. In the Armington model, this 100 units is a C.E.S. aggregate of 75 units of good i produced by Home and 25 units of good i imported from Foreign. As illustrated by Figure 2, preferences over the aggregate goods i and j, and preferences over the lower-level varieties produced by Home and Foreign are represented by C.E.S. functions. We presume that the upper-level substitution elasticity is equal to $\sigma = 1$, and that the lower-level substitution elasticities are equal to $\sigma_i = 4$.

By setting the upper-level C.E.S. substitution elasticity equal to $\sigma = 1$ (upper-level utility function is Cobb-Douglas), we are effectively assuming that the demand elasticity for any of the upper-level aggregate goods is equal to $\epsilon_i = -\sigma - (1-\sigma)S_i = -1$. This implies that the lower-level demand elasticity will be independent of σ, since from equation (3), $\frac{\partial z_{ir}}{\partial p_{ir}}\frac{p_{ir}}{z_{ir}} = \epsilon_{ir} = -\sigma_i - S_{ir}(1-\sigma_i) + S_{ir}(1+\epsilon_i),\ r = d, m$.

It is worth noting how this version of the Armington model differs from that employed in a typical large-scale CGE trade model like the Global Trade Analysis Project (GTAP).[11] Of course, the level of aggregation in a large-scale CGE model is often considerably finer than that employed here. The latest version of the GTAP model allows commodity disaggregation to 57 goods and 87 regions, though CGE applications of the GTAP model typically aggregate together sectors/regions which are not of particular

[11]See Hertel (1997) for a complete description of the GTAP model.

	Evaluated at initial equilibrium				Evaluated at optimal import tariff			
	supply elasticities				supply elasticities			
	Home		Foreign		Home		Foreign	
	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2
1	0.0000		0.0000		0.0000		0.0000	
2		0.0000		0.0000		0.0000		0.0000
	uncompensated demand elasticities				uncompensated demand elasticities			
	Home		Foreign		Home		Foreign	
	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2
1	-2.0000	1.0000	-2.5000	1.5000	-1.7074	0.7074	-3.0202	2.0202
2	2.0000	-3.0000	1.5000	-2.5000	2.2926	-3.2926	0.9798	-1.9798
	compensated demand elasticities				compensated demand elasticities			
	Home		Foreign		Home		Foreign	
	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2
1	-1.3333	1.3333	-2.0000	2.0000	-0.9431	0.9431	-2.6936	2.6936
2	2.6667	-2.6667	2.0000	-2.0000	3.0569	-3.0569	1.3064	-1.3064
	uncompensated trade elasticities				uncompensated trade elasticities			
	Home		Foreign		Home		Foreign	
	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2
1	4.0000	-2.0000	-2.5000	1.5000	8.2731	-3.4275	-3.0202	2.0202
2	2.0000	-3.0000	-1.5000	2.5000	2.2926	-3.2926	-2.0202	4.0820
	compensated trade elasticities				compensated trade elasticities			
	Home		Foreign		Home		Foreign	
	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2	Good 1	Good 2
1	2.6667	-2.6667	-2.0000	2.0000	4.5700	-4.5700	-2.6936	2.6936
2	2.6667	-2.6667	-2.0000	2.0000	3.0569	-3.0569	-2.6936	2.6936
	offer curve elasticity				offer curve elasticity			
	Home		Foreign		Home		Foreign	
	1.5000000		1.6666667		1.40212510		1.49500149	
	import demand elasticity				import demand elasticity			
	Home		Foreign		Home		Foreign	
	3.0000000		2.5000000		3.48678829		3.02019593	

Table 5: Elasticities in One-Sector Armington Exchange Economy

Home	good i	good j	consump	export	import	demand
good i			100	75	25	150
good j			100	25	75	50
labour	100	20				
capital	50	30				
supply	150	50				

Foreign	good i	good j	consump	export	import	demand
good i			100	25	75	50
good j			100	75	25	150
labour	33.3	60				
capital	16.7	90				
supply	50	150				

Table 6: Social Accounting Matrix for two-good two-factor Armington Economy

concern. The nesting method used here to represent consumption and trade is the same as that used in CGE models which use the Armington assumption to model trade in differentiated products, except that the lower-level import good is typically disaggregated by source. For example, suppose Home imports good i from a number of different countries. In our model, this would be represented by a third nest below "Imported i" in Figure 1 which disaggregates imports by country. Substitution between imports from different sources would be governed by a C.E.S. substitution elasticity σ_{ir}. It is typical in large-scale CGE model with this type of nesting structure to set the lowest level C.E.S. substitution elasticities equal to $\sigma_{ir} = 2 \cdot \sigma_i$.[12] But it is importnat to note that our earlier result is just as relevant here: If we choose σ_i to calibrate our model to an econometrically estimated import demand elasticity, use of the C.E.S. function implies a value for the demand elasticity for the domestically produced substitute which will generally not be consistent with any econometrically estimated values. Indeed, CGE models typically do not report the values of the domestic demand elasticities which are implied by the calibration process.

To illustrate, elasticities consistent with the initial general equilibrium in our simple two-good two-factor two-country Armington model are given in Table 7. If our econometric estimates of Home's (uncompensated) elasticity of demand for imports of goods 1 and 2 from Foreign were $\epsilon_{1m} = -3.25$ and $\epsilon_{2m} = -1.75$, respectively, then we would set $\sigma_1 = \sigma_2 = 4$ to calibrate our model.[13] This implies that the own-price demand

[12] See Liu *et al* (2004) for empirical support for this *rule of two.*

[13] Rows in Table 7 identify the good, while columns refer to prices. So columns headed 'Good H' contain the elasticity with respect to Home's price of good 1 (2) for rows 1H or 1F (2H or 2F). Columns

elasticities for Home's domestically produced varieties of goods 1 and 2 are $\epsilon_{1d} = -1.75$ and $\epsilon_{2d} = -3.25$, respectively.[14]

Now suppose the Home country charges an optimal tariff on imports of good 1 from Foreign. Given the SAM in Table 6 and the independently specified C.E.S. substitution elasticities in production, consumption and trade, Home will charge an optimal tariff of $t = 0.247$ or 24.7%. Elasticities consistent with Home charging this optimal tariff are reported on the right-hand side of Table 7. Again, it is clear that these elasticities are not constant, and that the C.E.S. substitution elasticity $\sigma_i = 4$ is a poor approximation of the import demand elasticity. For example, Home's uncompensated (compensated) elasticity of demand for imports of good 1 from Foreign rises (in absolute value) from -3.25 (-3.125) in the initial equilibrium to -3.4379 (-3.3442) when Home charges an optimal tariff on imports of good 1 from Foreign.

Now we repeat this experiment using the same initial data set to represent the initial trading equilibrium, but allowing output supplies to adjust. We assume both labour and capital are perfectly mobile between industries, and that production functions are Cobb-Douglas. Elasticities consistent with the initial general equilibrium in this simple two-good two-factor two-country Armington model are given in Table 8.

Home's optimal tariff on imports of good 1 from Foreign is $t = 0.360$ or 36.0%. In the optimal tariff equilibrium, Home's compensated import demand elasticity for imports of good 1 (2) from Foreign of -3.5144 (-1.2943) compares to the corresponding elasticities in the intial equilibrium of -3.1250 (-1.3750). Recall that the C.E.S. substitution elasticity in the lower-level nest is 4 throughout.

headed 'Good A' indicate an elasticity with respect to the price of the other aggregate good.

[14]Note that since the upper-level substitution elasticity is $\sigma = 1$ (upper-level utility function is Cobb-Douglas), (uncompensated) demand functions for any good are independent of the price of the other aggregate good, so the uncompensated cross-price demand elasticity with respect to the price of the other aggregate good is zero.

	Evaluated at initial equilibrium						Evaluated at optimal import tariff					
	supply elasticities						supply elasticities					
	Home		Foreign				Home		Foreign			
	Good 1	Good 2	Good 1	Good 2			Good 1	Good 2	Good 1	Good 2		
1	0.0000	0.0000	0.0000	0.0000			0.0000	0.0000	0.0000	0.0000		
2	0.0000	0.0000	0.0000	0.0000			0.0000	0.0000	0.0000	0.0000		
	uncompensated demand elasticities						uncompensated demand elasticities					
	Home			Foreign			Home			Foreign		
	Good H	Good A	Good F	Good H	Good A	Good F	Good H	Good A	Good F	Good H	Good A	Good F
1 H	-1.7500	0.0000	0.7500	-1.7500	0.0000	0.7500	-1.5621	0.0000	0.5621	-1.9262	0.0000	0.9262
1 F	2.2500	0.0000	-3.2500	2.2500	0.0000	-3.2500	2.4379	0.0000	-3.4379	2.0738	0.0000	-3.0738
2 H	-3.2500	0.0000	2.2500	-3.2500	0.0000	2.2500	-3.2500	0.0000	2.2500	-3.2500	0.0000	2.2500
2 F	0.7500	0.0000	-1.7500	0.7500	0.0000	-1.7500	0.7500	0.0000	-1.7500	0.7500	0.0000	-1.7500
	compensated demand elasticities						compensated demand elasticities					
	Home			Foreign			Home			Foreign		
	Good H	Good A	Good F	Good H	Good A	Good F	Good H	Good A	Good F	Good H	Good A	Good F
1 H	-1.3750	0.5000	0.8750	-1.3750	0.5000	0.8750	-1.1558	0.5000	0.6558	-1.5806	0.5000	1.0806
1 F	2.6250	0.5000	-3.1250	2.6250	0.5000	-3.1250	2.8442	0.5000	-3.3442	2.4194	0.5000	-2.9194
2 H	-3.1250	0.5000	2.6250	-3.1250	0.5000	2.6250	-3.1250	0.5000	2.6250	-3.1250	0.5000	2.6250
2 F	0.8750	0.5000	-1.3750	0.8750	0.5000	-1.3750	0.8750	0.5000	-1.3750	0.8750	0.5000	-1.3750
	uncompensated trade elasticities						uncompensated trade elasticities					
	Home			Foreign			Home			Foreign		
	Good H	Good A	Good F	Good H	Good A	Good F	Good H	Good A	Good F	Good H	Good A	Good F
1 H	1.7500	0.0000	-0.7500	-1.7500	0.0000	0.7500	1.9199	0.0000	-0.6909	-1.9262	0.0000	0.9262
1 F	2.2500	0.0000	-3.2500	-2.2500	0.0000	3.2500	2.4379	0.0000	-3.4379	-4.0740	0.0000	6.0385
2 H	3.2500	0.0000	-2.2500	-3.2500	0.0000	2.2500	3.3978	0.0000	-2.3523	-3.2500	0.0000	2.2500
2 F	0.7500	0.0000	-1.7500	-0.7500	0.0000	1.7500	0.7500	0.0000	-1.7500	-0.7174	0.0000	1.6739
	compensated trade elasticities						compensated trade elasticities					
	Home			Foreign			Home			Foreign		
	Good H	Good A	Good F	Good H	Good A	Good F	Good H	Good A	Good F	Good H	Good A	Good F
1 H	1.3750	-0.5000	-0.8750	-1.3750	0.5000	0.8750	1.4205	-0.6145	-0.8060	-1.5806	0.5000	1.0806
1 F	2.6250	0.5000	-3.1250	-2.6250	-0.5000	3.1250	2.8442	0.5000	-3.3442	-4.7530	-0.9823	5.7353
2 H	3.1250	-0.5000	-2.6250	-3.1250	0.5000	2.6250	3.2671	-0.5227	-2.7444	-3.1250	0.5000	2.6250
2 F	0.8750	0.5000	-1.3750	-0.8750	-0.5000	1.3750	0.8750	0.5000	-1.3750	-0.8369	-0.4782	1.3152

Table 7: Elasticities in Two-Sector Armington Exchange Economy

Evaluated at initial equilibrium

supply elasticities

	Home		Foreign	
	Good 1	Good 2	Good 1	Good 2
1	4.2500	-4.2500	13.2500	-13.2500
2	-12.7500	12.7500	-4.4167	4.4167

uncompensated demand elasticities

	Home			Foreign		
	Good H	Good A	Good F	Good H	Good A	Good F
1 H	-1.7500	0.0000	0.7500	-1.7500	0.0000	0.7500
1 F	2.2500	0.0000	-3.2500	2.2500	0.0000	-3.2500
2 H	-3.2500	0.0000	2.2500	-3.2500	0.0000	2.2500
2 F	0.7500	0.0000	-1.7500	0.7500	0.0000	-1.7500

compensated demand elasticities

	Home			Foreign		
	Good H	Good A	Good F	Good H	Good A	Good F
1 H	-1.3750	0.5000	0.8750	-1.3750	0.5000	0.8750
1 F	2.6250	0.5000	-3.1250	2.6250	0.5000	-3.1250
2 H	-3.1250	0.5000	2.6250	-3.1250	0.5000	2.6250
2 F	0.8750	0.5000	-1.3750	0.8750	0.5000	-1.3750

uncompensated trade elasticities

	Home			Foreign		
	Good H	Good A	Good F	Good H	Good A	Good F
1 H	10.2500	-8.5000	-0.7500	-1.7500	0.0000	0.7500
1 F	2.2500	0.0000	-3.2500	-2.2500	-26.5000	29.7500
2 H	28.7500	-25.5000	-2.2500	-3.2500	0.0000	2.2500
2 F	0.7500	0.0000	-1.7500	-0.7500	-8.8333	10.5833

compensated trade elasticities

	Home			Foreign		
	Good H	Good A	Good F	Good H	Good A	Good F
1 H	9.8750	-9.0000	-0.8750	-1.3750	0.5000	0.8750
1 F	2.6250	0.5000	-3.1250	-2.6250	-27.0000	29.6250
2 H	28.6250	-26.0000	-2.6250	-3.1250	0.5000	2.6250
2 F	0.8750	0.5000	-1.3750	-0.8750	-9.3333	10.2083

Evaluated at optimal import tariff

supply elasticities

	Home		Foreign	
	Good 1	Good 2	Good 1	Good 2
1	4.0956	-4.0956	16.7225	-16.7225
2	-14.2408	14.2408	-4.1506	4.1506

uncompensated demand elasticities

	Home			Foreign		
	Good H	Good A	Good F	Good H	Good A	Good F
1 H	-1.4162	0.0000	0.4162	-1.8647	0.0000	0.8647
1 F	2.5838	0.0000	-3.5838	2.1353	0.0000	-3.1353
2 H	-3.3191	0.0000	2.3191	-3.3191	0.0000	2.3191
2 F	0.6809	0.0000	-1.6809	0.6809	0.0000	-1.6809

compensated demand elasticities

	Home			Foreign		
	Good H	Good A	Good F	Good H	Good A	Good F
1 H	-0.9856	0.5000	0.4856	-1.5088	0.5000	1.0088
1 F	3.0144	0.5000	-3.5144	2.4912	0.5000	-2.9912
2 H	-3.2057	0.5000	2.7057	-3.2057	0.5000	2.7057
2 F	0.7943	0.5000	-1.2943	0.7943	0.5000	-1.2943

uncompensated trade elasticities

	Home			Foreign		
	Good H	Good A	Good F	Good H	Good A	Good F
1 H	11.2501	-9.4118	-0.5403	-1.8647	0.0000	0.8647
1 F	2.5838	0.0000	-3.5838	-5.6229	-60.7583	69.0145
2 H	33.0775	-29.5170	-2.4878	-3.3191	0.0000	2.3191
2 F	0.6809	0.0000	-1.6809	-0.6347	-8.0200	9.5869

compensated trade elasticities

	Home			Foreign		
	Good H	Good A	Good F	Good H	Good A	Good F
1 H	10.6912	-10.0608	-0.6303	-1.5088	0.5000	1.0088
1 F	3.0144	0.5000	-3.5144	-6.5600	-62.0749	68.6349
2 H	32.9558	-30.0534	-2.9024	-3.2057	0.5000	2.7057
2 F	0.7943	0.5000	-1.2943	-0.7405	-8.4861	9.2266

Table 8: Elasticities in Two-Sector Armington Production Economy

3.3 Discussion and Extensions

In all models described thus far, the only use for imports has been for domestic consumption. It is typical for imports (as well as their domestically produced substitutes) to also be used as intermediate inputs in production.[15] In fact, for primary products and less-processed goods, usage by the production sector as intermediate inputs represents a larger share of consumption of imports than usage by domestic consumers. In models which accommodate usage of imports as intermediate inputs in production, the production technology is commonly modelled using a Leontief production function, where intermediate inputs are combined with an aggregate value added (itself a C.E.S. aggregate of primary inputs) in fixed coefficients. In such models, it is important to acknowledge that the upper-level substitution elasticity in production analogous to $\sigma = 1$ in our upper-level utility function is equal to zero, so using σ_i as an approximation for the import demand elasticity will be even more problematic.

As mentioned earlier, it is most common in large-scale CGE models to disaggregate imports by source. In a model with r regions, any individual region will import from $r - 1$ other regions. Imports from different sources is accommodated by a nest below the "Foreign" in Figure 1 with a C.E.S. substitution elasticity σ_{ir} governing substitution between imports from different sources. As should be clear from earlier discussions, this single substitution elasticity can be used to accurately calibrate the model to an import demand elasticity for only one of the $r - 1$ regions. Since these trade elasticities are so important in affecting the results of many trade policy shocks examined using CGE models, it seems important that these models give a more complete description of all of the trade elasticities to which the model is calibrated, not just the single C.E.S. substitution elasticity.

4 Conclusion

It is common to trade in CGE models by using the so-called Armington assumption which models the same good produced in different countries as a differentiated product. This has both desirable and undesirable implications: CGE models can easily accommodate cross-hauling, where the same good is both imported and exported, an important feature of trade data. But with differentiated products, each country retains some monopoly power over trade. In this case, it is important to model the process by which such CGE

[15] Large scale CGE datasets like GTAP also account for consumption by governments and consumption for investment.

models are calibrated to international trade elasticities as thoroughly as possible. In this paper we have presented some simple CGE models of trade, and detailed the complete set of demand and trade elasticities implied by the calibration process. This has made it clear that using nested C.E.S. functions to model trade allows a model to be able to be calibrated only to a single import demand elasticity, while the domestic demand elasticity corresponding to the same good is determined as a residual.

We have further demonstrated how import demand elasticities are only approximated by the exogenously specified C.E.S. substitution elasticities. The perils of using such approximations were illustrated by solving for a nation's optimal import tariff, which was related to trade elasticities through optimal tariff formulae.

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Appendix: Optimal tariff as a function of compensated trade elasticities

Begin with the Balance of Payments constraint: $p_m m(p_e,p_m) = p_e e(p_e,p_m) + t_m p_m^* m(p_e,p_m)$, where exports are defined as $e(p_e,p_m,v,\mu) = y^e(p_e,p_m,v) - z^e(p_e,p_m,\mu)$, imports as $m(p_e,p_m,v,\mu) = z^m(p_e,p_m,\mu) - y^m(p_e,p_m,v)$, and y^i and z^i are the output supply and compensated demand functions, respectively. Take a derivative with respect to the import tariff t_m, which if chosen optimally, will impy that $\partial\mu/\partial t_m = 0$ (factor endowments v are assumed to remain constant):

$$m\frac{\partial p_m}{\partial t_m} + p_m\frac{\partial m}{\partial p_e}\frac{\partial p_e}{\partial t_m} + p_m\frac{\partial m}{\partial p_m}\frac{\partial p_m}{\partial t_m} \;=\; e\frac{\partial p_e}{\partial t_m} + p_e\frac{\partial e}{\partial p_e}\frac{\partial p_e}{\partial t_m} + p_e\frac{\partial e}{\partial p_m}\frac{\partial p_m}{\partial t_m} + p_m^* m + t_m m\frac{\partial p_m^*}{\partial t_m} + t_m p_m^*\frac{\partial m}{\partial p_e}\frac{\partial p_e}{\partial t_m} + t_m p_m^*\frac{\partial m}{\partial p_m}\frac{\partial p_m}{\partial t_m}$$

Substitute away for $\frac{\partial p_m}{\partial t_m} = \frac{\partial p_m^*}{\partial t_m} + p_m^* + t_m\frac{\partial p_m^*}{\partial t_m}$:

$$\begin{aligned} m\frac{\partial p_m^*}{\partial t_m} + p_m^* m + m t_m\frac{\partial p_m^*}{\partial t_m} + p_m\frac{\partial m}{\partial p_e}\frac{\partial p_e}{\partial t_m} + p_m\frac{\partial m}{\partial p_m}\frac{\partial p_m^*}{\partial t_m} + p_m p_m^*\frac{\partial m}{\partial p_m} + t_m p_m\frac{\partial m}{\partial p_m}\frac{\partial p_m^*}{\partial t_m} = e\frac{\partial p_e}{\partial t_m} + p_e\frac{\partial e}{\partial p_e}\frac{\partial p_e}{\partial t_m} + p_e\frac{\partial e}{\partial p_m}\frac{\partial p_m^*}{\partial t_m} + p_e p_m^*\frac{\partial e}{\partial p_m} \\ +\;\; t_m p_e\frac{\partial e}{\partial p_m}\frac{\partial p_m^*}{\partial t_m} + p_m^* m + m t_m\frac{\partial p_m^*}{\partial t_m} + t_m p_m^*\frac{\partial m}{\partial p_e}\frac{\partial p_e}{\partial t_m} + t_m p_m^*\frac{\partial m}{\partial p_m}\frac{\partial p_m^*}{\partial t_m} + t_m p_m^* p_m^*\frac{\partial m}{\partial p_m} + t_m t_m p_m^*\frac{\partial m}{\partial p_m}\frac{\partial p_m^*}{\partial t_m} \end{aligned}$$

Cancel terms, and collect the final 4 terms on the left-hand-side and the right-hand-side, noting that $p_m - t_m p_m^* = p_m^*$:

$$m\frac{\partial p_m^*}{\partial t_m} + p_m^*\frac{\partial m}{\partial p_e}\frac{\partial p_e}{\partial t_m} + p_m^*\frac{\partial m}{\partial p_m}\frac{\partial p_m^*}{\partial t_m} + p_m^* p_m^*\frac{\partial m}{\partial p_m} + t_m p_m^*\frac{\partial m}{\partial p_m}\frac{\partial p_m^*}{\partial t_m} \;=\; e\frac{\partial p_e}{\partial t_m} + p_e\frac{\partial e}{\partial p_e}\frac{\partial p_e}{\partial t_m} + p_e\frac{\partial e}{\partial p_m}\frac{\partial p_m^*}{\partial t_m} + p_e p_m^*\frac{\partial e}{\partial p_m} + t_m p_e\frac{\partial e}{\partial p_m}\frac{\partial p_m^*}{\partial t_m}$$

Re-write derivative terms as elasticities. Since exports are not taxed, $p_e = p_e^*$:

$$\left[m + \frac{p_m^* m}{p_m}\epsilon_{mm} + \frac{t_m p_m^* m}{p_m}\epsilon_{mm} - \frac{e p_e^*}{p_m}\epsilon_{em} - \frac{t_m p_e^* e}{p_m}\epsilon_{em}\right]\frac{\partial p_m^*}{\partial t_m} - \left[e + e\epsilon_{ee} - \frac{p_m^* m}{p_e^*}\epsilon_{me}\right]\frac{\partial p_e^*}{\partial t_m} \;=\; p_m^*\frac{p_e^* e}{p_m}\epsilon_{em} - p_m^*\frac{p_m^* m}{p_m}\epsilon_{mm}$$

Collect terms in the first square brackets:

$$\left[m + \frac{1+t_m}{p_m} p_m^* m \epsilon_{mm} - \frac{1+t_m}{p_m} e p_e^* \epsilon_{em}\right] \frac{\partial p_m^*}{\partial t_m} - \left[e + e\epsilon_{ee} - \frac{m p_m^*}{e p_e^*} e \epsilon_{me}\right] \frac{\partial p_e^*}{\partial t_m} = \frac{p_m^*}{p_m} \left[e p_e^* \epsilon_{em} - m p_m^* \epsilon_{mm}\right]$$

Note that $\frac{1+t_m}{p_m} = \frac{1}{p_m^*}$:

$$m \left[1 + \epsilon_{mm} - \frac{e p_e^*}{m p_m^*} \epsilon_{em}\right] \frac{\partial p_m^*}{\partial t_m} - e \left[1 + \epsilon_{ee} - \frac{m p_m^*}{e p_e^*} \epsilon_{me}\right] \frac{\partial p_e^*}{\partial t_m} = \frac{p_m^*}{p_m} \left[e p_e^* \epsilon_{em} - m p_m^* \epsilon_{mm}\right]$$

The value of exports at world prices must equal the value of imports at world prices, so $\frac{e p_e^*}{m p_m^*} = 1$:

$$m \left[1 + \epsilon_{mm} - \epsilon_{em}\right] \frac{\partial p_m^*}{\partial t_m} - e \left[1 + \epsilon_{ee} - \epsilon_{me}\right] \frac{\partial p_e^*}{\partial t_m} = \frac{p_m^*}{p_m} \left[e p_e^* \epsilon_{em} - m p_m^* \epsilon_{mm}\right]$$

Multiply both sides by $\frac{p_m}{p_m^*} = 1 + t_m$, and multiply and divide each term on the left-hand-side by p_m^*:

$$\frac{1+t_m}{p_m^*} \left\{ m p_m^* \left[1 + \epsilon_{mm} - \epsilon_{em}\right] \frac{\partial p_m^*}{\partial t_m} - \frac{p_m^*}{p_e^*} e p_e^* \left[1 + \epsilon_{ee} - \epsilon_{me}\right] \frac{\partial p_e^*}{\partial t_m} \right\} = e p_e^* \epsilon_{em} - m p_m^* \epsilon_{mm}$$

Divide each term by $e p_e^* = m p_m^*$:

$$\frac{1+t_m}{p_m^*} \left\{ \left[1 + \epsilon_{mm} - \epsilon_{em}\right] \frac{\partial p_m^*}{\partial t_m} - \frac{p_m^*}{p_e^*} \left[1 + \epsilon_{ee} - \epsilon_{me}\right] \frac{\partial p_e^*}{\partial t_m} \right\} = \epsilon_{em} - \epsilon_{mm}$$

$$\frac{1+t_m}{t_m} \left\{ \left[1 + \epsilon_{mm} - \epsilon_{em}\right] \frac{\partial p_m^*/p_m^*}{\partial t_m/t_m} - \left[1 + \epsilon_{ee} - \epsilon_{me}\right] \frac{\partial p_e^*/p_e^*}{\partial t_m/t_m} \right\} = \epsilon_{em} - \epsilon_{mm}$$

We can choose one price as numeraire, so suppose $\frac{\partial p_e^*/p_e^*}{\partial t_m/t_m} = 0$. This allows us to write the optimal tariff as:

$$t_m = \frac{\left[1 + \epsilon_{mm} - \epsilon_{em}\right] \frac{\partial p_m^*/p_m^*}{\partial t_m/t_m}}{\epsilon_{em} - \epsilon_{mm} - \left[1 + \epsilon_{mm} - \epsilon_{em}\right] \frac{\partial p_m^*/p_m^*}{\partial t_m/t_m}}$$