Trade and Variety in a Model with Endogenous Product Differentiation

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Abstract

This paper sets up a model of endogenous product differentiation to analyze the variety effects of international trade. In our model multi-product firms decide not only about the number of varieties they supply but also about the degree of horizontal differentiation between these varieties. Firms can raise the degree of differentiation by investing variety specific fixed costs. In this setting, we analyze how trade integration, i.e. an increase in market size, influences the number of firms in the market, the number of products per firm and the degree of differentiation.

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1 Introduction

If economists talk about the effects of international trade on product variety, they typically have in mind the influence of trade on the number of heterogeneous goods. Variety, however, can be much broader than this. In addition to the mere number of goods it also encompasses their specification. Heterogeneous goods may differ only slightly from each other – for example just in their color – or more substantially with respect to one or several more important characteristics. Our analysis starts from this observation and endogenizes product differentiation in an otherwise standard "love of variety" model of intra-industry trade. We consider multi-product firms that decide not only about the number of varieties but also about the degree of horizontal product differentiation between these varieties. The equilibrium degree of product differentiation emerges from a trade-off between positive demand effects and additional product specific fixed costs. International trade may then influence variety through three different channels: by changing *(i)* the number of firms on the market, *(ii)* the number of varieties firms supply, and *(iii)* the degree of product differentiation between these varieties.

Our model of endogenous product differentiation produces a couple of interesting insights: Firstly, the activities of firms to differentiate their products may have consequences for market entry and vice versa. On the one hand, an increase in the degree of product differentiation lowers the number of firms in the market; on the other hand an increase in the number of firms diminishes the incentives for product differentiation. Secondly, under certain conditions the effects of trade on product differentiation may be qualitatively different to the effects on the number of products. More specifically, in the baseline specification of our model trade integration raises the degree of product differentiation whereas the number of products per firm remains constant and the number of firms in the market may rise or decline. Thus we can have a situation, in which the total number of products declines but which is nevertheless characterized by an increase in product differentiation. Just counting the number of firms and goods would then yield a misleading picture of the variety effects of international trade. Thirdly, firms may regard the number of goods and the degree of product differentiation as complementary instruments. An increase in the degree of product differentiation then makes it profitable to supply additional product varieties. We obtain this last result in a variation of our baseline model with a slightly different cost function.

The theoretical literature on international trade has recently become increasingly engaged in analyzing the behavior of multi-product firms.[1] For example, Feenstra and Ma (2007) consider multi-product firms in a Dixit-Stiglitz model of intra-industry trade. In their paper, firms balance the additional sales generated from introducing new product varieties against a "cannibalization" of sales of their existing varieties. From this trade-off, Feenstra and Ma (2007) determine the equilibrium number of varieties per firm. They assume a constant elasticity of substitution between varieties, and firms can not influence the degree of product differentiation. Allanson and Montagna (2005) consider a nested CES-utility with a lower elasticity of substitution between individual varieties supplied by a single firm compared to varieties supplied by different firms. In their approach it makes a difference for total variety whether firms raise the number of products or the number of firms increases. However, Allanson and Montagna (2005) also do not allow for endogenous product differentiation.[2] In our model firms can determine how strong cannibalization is by deciding about product differentiation.

Since we assume that product differentiation influences fixed costs, our model is somewhat related to the endogenous sunk-cost literature.[3] According to this literature firms can raise product quality or reduce variable costs by increasing fixed cost investments upon market entry. Shaked and Sutton (1987) show that under certain conditions a lower bound for the market share of at least one firm exists if fixed costs are endogenous. As market size grows, at least one firm invests more to retain a certain market share. Eckel (2006b) incorporates endogenous sunk costs in an otherwise standard model of intra-industry trade with one product per firm. He determines conditions under which the number of firms increases or decreases with market integration. We show that this result of a possible decline in the number of firms carries over to the case of multi-product firms and endogenous product differentiation. In our set-up, however, a decline in the number of firms is not equivalent with lower variety as firms raise the degree of product differentiation.

An alternative to CES-preferences are models of spatial product differentiation, as for example with flexible manufacturing.[4] According to the

[1] In addition to the papers discussed below, see e.g. Brambilla (2006), Baldwin and Gu (2006), Bernard, Redding, and Schott (2006), and Nocke and Yeaple (2006).

[2] In addition, Allanson and Montagna (2005) neglect the cannibalization effect by assuming that firms are small. Erkel-Rousse (1997) considers vertical product differentiation with multi-product firms and CES-preferences.

[3] See e.g. Dasgupta and Stiglitz (1980), Shaked and Sutton (1984, 1987), Sutton (1991, 1998, or 2007). Kshirsagar (2006) considers a heterogeneous firm model as in Melitz (2003) with endogenous sunk-costs.

[4] Yet another set-up is chosen by Anderson and de Palma (2003) who analyze the behavior of multi-product firms in a model with a nested logit demand function. Hansen and Jørgensen (2001) and Hansen and Nielsen (2007) consider a linear demand function;

flexible manufacturing approach firms cover a whole area around their "core competence" in a Salop-type circular market. Marginal costs increase with the distance from a firms' core competence, such that diseconomies of scope limit firms' expansion over the product space. Eckel (2006a) introduces endogenous sunk-costs into such a flexible manufacturing setting.[5] In his model firms do not influence the degree of product differentiation but can reduce variable costs by investing into R&D. He finds that R&D investments clearly increase in market size whereas the effects on the number and scope of firms are ambiguous. Our paper starts from a model with a CES-utility function as in Feenstra and Ma (2007).

The remainder of the paper is organized as follows: Section 2 presents the baseline model. In section 3 we analyze the equilibrium and derive the effects of market size on product differentiation – first for a given number of firms and then with free market entry. In section 4 we consider a different costs function which results in a more complex picture of the relationship between product differentiation, the number of products and firms.

2 The Model

We consider a market with m symmetric firms ($i = 1 \ldots m$). Each of these firms supplies a continuum of differentiated varieties with mass n (the "number of varieties"). Free market entry determines m endogenously. On the demand side we assume L households with income normalized to 1. This assumption is compatible with general equilibrium if we assume a constant labor productivity of 1 and zero profits. Households consume a differentiated manufacturing aggregate C and an agricultural good A, the numèraire. Preferences with respect to these two goods are Cobb-Douglas, such that households spend a constant share μ of their income for C and $1 - \mu$ for A. A CES-function specifies the manufacturing aggregate:

$$C = \left(\sum_{i=1}^{m} \tilde{c}_i^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)} . \qquad (1)$$

in their model production strategies of multi-product firms are determined by the influence of the number of goods or the number of plants on fixed and variable costs. Blanchard, Gaigne, and Mathieu (2007) analyze product differentiation in a Hotelling-model with a linear market. Doraszelski and Draganska (2006) finally analyze product differentiation strategies in a duopoly by assuming that firms can either produce general purpose products or products that are targeted to a certain market segment.

[5]For other recent papers on flexible manufacturing see e.g. Eaton and Schmitt (1994), Norman and Thisse (1999), or Eckel and Neary (2006).

In (1) the sub-utility $\tilde{c}_i$ is an aggregate of all varieties supplied by firm i. We assume perfect symmetry between varieties of firm i, and we allow for a variable degree of differentiation between these varieties. More specifically, we assume the following specification of the sub-utility:

$$\tilde{c}_i = v_i \left(\int_0^{n_i} c_i(k)^{(\sigma-1)/\sigma} dk \right)^{\sigma/(\sigma-1)} + (1 - v_i) \int_0^{n_i} c_i(k) dk \ .$$

Given our symmetry assumption, we can set $c_i(k) = c_i$ for all k to obtain

$$\tilde{c}_i = \left[v n_i^{\sigma/(\sigma-1)} + (1 - v) n_i \right] c_i \ . \quad (2)$$

The term v_i is the degree of differentiation between varieties ($0 \leq v_i \leq 1$). To interpret v_i, first consider the limit case $v = 0$. In this case all varieties of firm i are perfect substitutes and $\tilde{c}_i = n_i c_i$. The aggregate of firm i's varieties is just equal to the total quantity produced, and our model then does not differ from a standard Dixit-Stiglitz setting with one product per firm. In the other extreme $v_i = 1$ and $\tilde{c}_i = n_i^{(\sigma-1)/\sigma} c_i$. In this case consumers can substitute different varieties supplied by firm i in the same way as varieties supplied by different firms. This is the setting analyzed by Feenstra and Ma (2007). Our paper focuses on the intermediate cases in which $0 < v_i < 1$.

We can define $x_i \equiv \tilde{c}_i/(c_i n_i)$ as the sub-utility generated per unit of output by firm i. Inserting yields:

$$x_i = v_i n_i^{1/(\sigma-1)} + 1 - v \ .$$

This expression monotonically increases in n_i and in v_i. An increase in the number of varieties and in the degree of product differentiation makes the product range of firm i more attractive for consumers. We also have $\partial x_i^2/\partial n \partial v$. The marginal utility from raising the number of varieties increases in the degree of product differentiation.

Utility maximization of consumers leads to an individual demand of

$$\tilde{c}_i = \mu \tilde{p}_i^{-\sigma} \left(\sum_{j=1}^{m} \tilde{p}_j^{1-\sigma} \right)^{-1} \ , \quad (3)$$

where $\tilde{p}_i$ is the price index for the variety range supplied by firm i, i.e. $\tilde{p}_i \tilde{c}_i = n_i p_i c_i$ or $\tilde{p}_i = p_i/x_i$. For aggregate demand $Q_i = L n_i c_i$ we can write

$$Q_i = \mu L x_i^{\sigma-1} p_i^{-\sigma} \left(\sum_{j=1}^{m} x_j^{\sigma-1} p_j^{1-\sigma} \right)^{-1} \ .$$

Firms decide about the number of varieties n_i, the degree of product differentiation between these varieties v_i and their price p_i. With marginal costs of ϕ and fixed costs of F, profits of firm i are $\pi_i = (p_i - \phi)\,Q_i - F$. Product differentiation causes additional costs to the firm. We consider these costs by assuming variety specific fixed costs of gnv^θ, with $\theta > 1$. These costs increase in the degree of differentiation v between varieties and in the number of varieties n. In addition, there are firm-specific fixed costs of f that are independent of the number of varieties or the degree of product differentiation. Total fixed costs are then $F = f + gnv^\theta$.

3 Equilibrium and Trade Integration

This section characterizes the equilibrium of our model. From this equilibrium we can determine the effects of trade integration by increasing the size of the relevant market L. For illustrative purposes we proceed in two steps. First, section 3.1 deals with the (hypothetical) case of a given number of firms m. We then turn to the complete model with an endogenous number of firms and free market entry in section 3.2.

3.1 Given Number of Firms

We consider a market with at least 2 active firms, i.e. $m \geq 2$. Assuming symmetry of the competitors $-i$ we can write the profit of firm i as follows:

$$\pi_i = \mu L \frac{p_i - \phi}{p_i} \frac{x_i^{\sigma-1} p_i^{1-\sigma}}{x_i^{\sigma-1} p_i^{1-\sigma} + (m-1) x_{-i}^{\sigma-1} p_{-i}^{1-\sigma}} - f - gnv^\theta \ . \tag{4}$$

With a given number of firms this profit may differ from zero.[6]
From the first order condition for p_i we obtain for the mark-up in the symmetric equilibrium:

$$\frac{p - \phi}{p} = \frac{m}{1 + (m-1)\sigma} \ . \tag{5}$$

According to (5), the equilibrium mark-up depends on the number of firms. If there are more firms in the market, demand becomes more elastic and as a consequence firms reduce their mark-up.[7]

[6]By assuming a given income to determine demand, we neglect the influence of firms' profits on demand for the differentiated good. As a matter of fact, we switch to a partial equilibrium analysis here.

[7]This effect is similar to Feenstra and Ma (2007).

The first order conditions for n_i and v_i can be written as

$$\mu L \cdot \frac{p-\phi}{p} \cdot \frac{m-1}{m^2} \cdot \frac{vn^{1/(\sigma-1)}}{nx} = gv^{\theta} \; , \tag{6}$$

$$\mu L \cdot \frac{p-\phi}{p} \cdot \frac{m-1}{m^2} \cdot \frac{(\sigma-1)\left(n^{1/(\sigma-1)}-1\right)}{x} = gn\theta v^{\theta-1} \; . \tag{7}$$

From these two equations we can derive the equilibrium number of varieties per firm:

$$n = \left(\frac{\sigma-1}{\sigma-1-\theta}\right)^{\sigma-1} \; . \tag{8}$$

To make sure that the solution implies $n > 1$ we assume $\sigma > 1+\theta$.[8] According to (8) the equilibrium number of varieties is constant and does not change with market size or with the number of firms in the market. This property of our model, which substantially facilitates the analysis in this section, is due to our assumed cost function and does not necessarily carry over to other specifications. In section 4 we turn to an alternative setting in which the optimal number of varieties increases in n.

For the equilibrium degree of product differentiation v we obtain from the first-order conditions:

$$v^{\theta-1}\left[v(1-z)+z\right] n = \psi(m,L) \; , \qquad \text{where}$$

$$z = n^{1/(1-\sigma)} < 1 \qquad \text{and} \qquad \psi(m,L) = \frac{\mu L(m-1)(p-\phi)}{m^2 pg} \; . \tag{9}$$

According to (9) the equilibrium degree of product differentiation v depends on the market size L and on the number of firms m. For a given number of firms, trade integration causes an increase in the degree of product differentiation. The influence of market size on v is

$$\left.\frac{dv}{dL}\right|_{dm=0} = \frac{\psi_L}{nv^{\theta-2}\left[(\theta-1)(v(1-z)+z)+(1-z)v\right]} > 0 \; ,$$

as $\psi_L > 0$. For a given number of firms, product differentiation becomes more profitable as market size grows and consequently the equilibrium v increases.

An increase in the number of firms has the following effect on v:

$$\frac{dv}{dm} = \frac{\psi_m}{nv^{\theta-2}\left[(\theta-1)(v(1-z)+z)+(1-z)v\right]} \; .$$

[8] Given our previous assumption of $\theta > 1$, this requires $\sigma > 2$.

By inserting (5) into ψ and taking the first derivative we can show that $m \geq 2$ is sufficient for $\psi_m < 0$ for . Then an increase in the number of firms lowers the degree of product differentiation v. This result can be explained from two effects of m on the decision of firms: Firstly, according to (9) the mark-up declines if the the number of firms increases. This makes it less profitable to raise demand by product differentiation. Secondly, for a given mark-up the marginal influence of product differentiation on individual sales declines in the number of competitors if $m > 2$.

3.2 Endogenous Number of Firms

This section endogenizes the number of firms by considering the case of free market entry. With an endogenous number of firms the size of the market may influence product variation not only directly – as in the previous section – but also indirectly by changing the number of firms that enter the market. With (5) the zero profit condition is:

$$\frac{\mu L}{1+(m-1)\sigma} = f + gnv^{\theta} \; . \tag{10}$$

Condition (10) determines m as a function of the market size L and of the degree of product differentiation v. For a given v an increase in L raises the number of firms in equilibrium:

$$\left.\frac{dm}{dL}\right|_{dv=0} = \frac{1+(m-1)\sigma}{\sigma L} > 0 \; .$$

An increase in the degree of product differentiation v lowers m:

$$\frac{dm}{dv} = -\frac{gn\theta v^{\theta-1}(1+(m-1)\sigma)^2}{\mu L} < 0 \; .$$

The more differentiated products are, the lower is the number of firms in the market.

As the degree of product differentiation and the number of firms are interdependent, we have to consider (9) and (10) simultaneously. Equations (9) and (10) can be written as

$$b(m)B(m)\mu L - a(v,z)A(v,n,g) = 0 \; , \tag{11}$$

$$B(m)\mu L - A(v,n,g) - f = 0 \; , \tag{12}$$

where $A(v,n,g) = gnv^{\theta}$, $a(v,z) = (v+z(1-v))v^{-1}$, $B(m) = (1+(m-1)\sigma)^{-1}$ and $b(m) = (m-1)m^{-1}$. For the relevant partial derivatives we have $A_v > 0$,

$A_g > 0$, $a_v = -zv^{-2} < 0$, $b' = m^{-2} > 0$, and $B' = -\sigma B^2 < 0$. Totally differentiating (11) and (12) leads to:

$$\begin{pmatrix} -(aA_v + a_v A) & (B'b + Bb')\,\mu L \\ -A_v & B'\mu L \end{pmatrix} \begin{pmatrix} dv \\ dm \end{pmatrix} = \begin{pmatrix} -bB\mu dL + aA_g dg \\ -B\mu dL + A_g dg + df \end{pmatrix} . \tag{13}$$

The term $-(aA_v + a_v A)$ is negative because of

$$aA_v + a_v A = [\theta v(1-z) + z(\theta - 1)]\, Av^{-2} > 0 .$$

If all firms raise v, the marginal profit of each firm declines. $B'\mu L$ is also negative. Entry by additional firms reduces profits per firm. For the respective sign of the other elements of the Jacobian matrix we have $A_v > 0$ and $(B'b + Bb')\,\mu L = [1 - \sigma(m-1)^2]\, B^2 m^{-2}\mu L > 0$. The Jacobian determinant can be written as:

$$|J| = (b-a)A_v B'\mu L + BA_v m^{-2}\mu L + zAB'v^{-2}\mu L .$$

Since $b < 1$, $a > 1$, $A_v > 0$ and $B' < 0$, the first two terms of this expression are positive; the third term, however, is negative such that the total sign of $|J|$ can be positive or negative. To obtain meaningful comparative static results, we have to assume dynamic stability, i.e. a positive sign of $|J|$. We do this for the following analysis.

As a first comparative static exercise, we may derive the influence of the firm specific fixed costs f on the model outcome. Not surprisingly, an increase in f lowers the number of firms in the market. Applying Cramer's rule yields:

$$\frac{dm}{df} = -\frac{A_v a + a_v A}{|J|} < 0 . \tag{14}$$

The degree of product differentiation increases in the level of firm specific sunk costs:

$$\frac{dv}{df} = -\frac{(B'b + Bb')\,\mu L}{|J|} > 0 . \tag{15}$$

This result follows from the influence of f on the number of firms in the market. An increase in f lowers the number of firms and thereby makes it more profitable for an individual firms to raise product differentiation. The effects of an increase in the variety specific fixed costs g are

$$\frac{dm}{dg} = -\frac{a_v A A_g}{|J|} > 0 , \qquad \text{and} \tag{16}$$

$$\frac{dv}{dg} = \frac{A_g \mu L\,[(a-b)\,B' - Bb']}{|J|} < 0 . \tag{17}$$

The product specific fixed costs g have contrary effects on the market outcome as the firm specific fixed costs f. Firms reduce product differentiation and as a consequence more firms enter the market if g increases.

Proposition 1 *Suppose fixed costs are $F = f + gnv^{\theta}$. An increase in f then lowers the number of firms in the market and raises the degree of product differentiation. An increase in g raises the number of firms in the market and lowers the degree of product differentiation. The number of goods per firm remains constant.*

To analyze the effects of trade integration we consider again an increase in the market size. The degree of product differentiation rises in L:

$$\frac{dv}{dL} = \frac{B^2 b' \mu^2 L}{|J|} > 0 \; . \tag{18}$$

The influence of L on the number of firms m can not be clearly signed. We have

$$\frac{dm}{dL} = \frac{(a-b)\, A_v B\mu + a_v A B \mu}{|J|} \; . \tag{19}$$

Here we have two effects working in opposite directions, on the one hand a positive direct effect of L on m but on the other hand an increase in product differentiation, which reduces the number of firms.

Figures 1 and 2 illustrate the effects of trade integration on product differentiation and on the number of firms. The line $m(v)$ depicts the first-order condition (9) for v. This line is downward sloping as a higher number of competitors lowers the marginal profit of raising v for each firm. The line $m(v)$ is the zero profit condition (8). This line has a negative slope as well, and dynamic stability requires that it is steeper than $v(m)$. An increase in market size shifts both lines upward. In figure 1 the degree of differentiation and the number of firms increases; in figure 2 the number of firms declines.

Although the influence of trade integration on the total number of firms is indeterminate, the number of firms per country clearly declines with an increase in the market size. This follows from the derivative

$$\frac{dm/L}{dL} = (\eta_{m,L} - 1)\,\frac{m}{L^2} \; ,$$

where $\eta_{m,L}$ is the elasticity of m with respect to L. From dm/dL we can derive

$$\eta_{m,L} = \frac{(a-b)\, A_v + a_v A}{[(a-b)\, A_v + a_v A]\, \sigma B m + A_v m^{-1}} < 1 \; ,$$

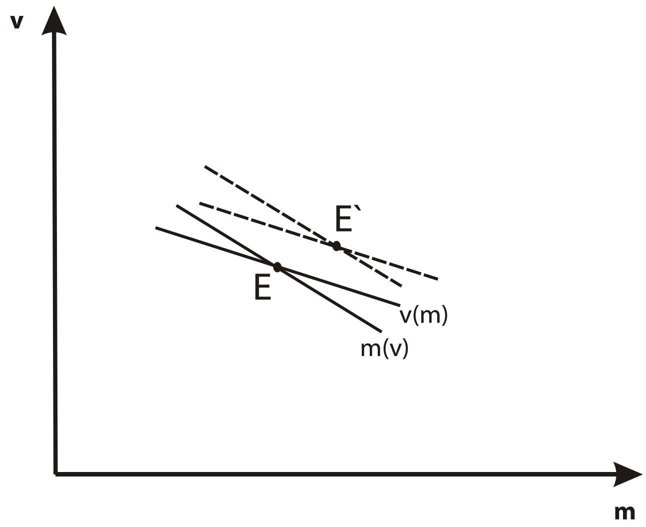

Figure 1: The Effects of Trade Integration: Case 1

because $\sigma Bm > 1$ and $A_v m^{-1} > 0$. The number of firms relative to L then declines in L. This implies that each firm produces on a larger scale. This result is similar to the findings of Feenstra and Ma (2007) for the case of a given degree of product differentiation and also to those of Krugman (1979) in his seminal analysis of intra-industry trade with monopolistic competition.

Proposition 2 *Suppose fixed costs are $F = f + gnv^{\theta}$. An increase in market size L then raises the degree of product differentiation. The total number of firms in the market may rise or decline, whereas the number of firms relative to L declines. The number of goods per firm remains constant.*

To determine the welfare effects of trade integration we define the aggregate price index as $\tilde{P} = \left(\sum_{j=1}^{m} \tilde{p}_j^{1-\sigma}\right)^{1/(1-\sigma)}$. Then $C = \mu/\tilde{P}$ and aggregate welfare declines in $\tilde{P}$. Inserting yields $\tilde{P} = (m)^{1/(1-\sigma)} px^{-1}$. Welfare depends on the number of firms, the price, and the sub-utility x, which in turn is determined by the number of varieties per firm and the degree of differentiation. We know already that x increases in the size of the market. From the mark-up formula (5) we obtain

$$p = \frac{1 + \sigma (m-1)}{(m-1)(\sigma - 1)} \phi \, .$$

The price p declines in the number of firms. Thus, if m increases or if the decline in m is sufficiently weak, then welfare clearly increases in L.

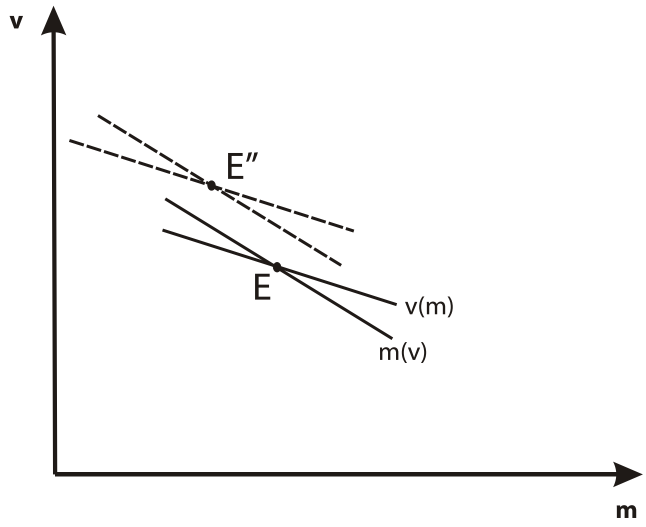

Figure 2: The Effects of Trade Integration: Case 2

4 Size-dependent number of varieties

So far, the profit maximizing number of varieties has been independent of the market size. Firms only adjust prices and the degree of differentiation to market size changes. In this section, we slightly change our specification of the cost function in order to make the number of varieties a more independent instrument: Fixed costs are now $f + gn\exp(\theta v)$ with $\sigma > 1 + \theta$ and $\theta > 1$. Marginal costs of the number of varieties are $g\exp(\theta v)$, marginal costs of variation are $n\theta$ times higher. The change of the cost function comes at the cost of less analytical tractability but provides additional insights as n now also changes with market conditions. The ratio of marginal costs with respect to the number of varieties and the degree of differentiation, $1/n\theta$, is now independent of v whereas the ratio of marginal returns is not (and both ratios depend on n), firms adjust both the number of varieties and the degree of differentiation to changes in cost parameters and in market size.

Proceeding first as in the previous sections, but replacing marginal costs of the number of varieties and marginal costs of variation on the right-hand sides of the first-order conditions (6) and (7), we obtain that the profit maximizing number of varieties per firm is

$$n = \left(\frac{\sigma - 1}{\sigma - 1 - \theta v} \right)^{\sigma - 1}. \qquad (20)$$

The optimum number of varieties and the degree of differentiation are positively correlated, i.e. $dn/dv = \theta n^{\sigma/(\sigma-1)} > 0$. An increase in the degree of differentiation raises the marginal return of the number of varieties relative to the marginal return of differentiation. Since the ratio of marginal costs stays unaltered, the number of varieties has to increase to balance marginal returns and marginal costs. An increases in the number of varieties reduces the ratio of marginal costs, but reduces the ratio of marginal returns even more. Note further that the mark-up pricing rule is not affected by the change in the specification of fixed costs.

The equilibrium number of firms

$$m = \frac{\sigma - 1}{\sigma} + \frac{L\mu}{\sigma[f + gn\exp(\theta v)]} \tag{21}$$

follows from the zero-profit condition. As can be seen from this condition, further differentiation and variation requires in equilibrium a lower number of varieties. Furthermore, provided that an increase in market size stimulates firms to raise the number of firms as well as the degree of differentiation, the influence on the equilibrium number of firms is still ambiguous – as it also turned out to be in the previous section.

We can rewrite the first-order conditions for m and v in the same way as in equations (11) and (12), with $\hat{A}(n, v, g) = gn\exp(\theta v)$ replacing $A(v, n, g)$ and $n = n(v)$ as determined in (20). Assuming again dynamic stability of the equilibrium we obtain the same equation (18) for the influence of L on the degree product differentiation. Because of (20) the number of varieties now also increases in L.

Proposition 3 *Suppose fixed costs are $F = f + gn\exp(\theta v)$. The degree of product differentiation and the number of goods per firm increase in the market size L.*

To obtain some more insights we proceed to numerical analysis. Using the mark-up pricing rule, equations (20) and (21), we can derive the free-entry equilibrium from the first-order condition wit respect to v. For benchmark parameters $\phi = 1$, $\theta = 2$, $g = 0.0001$, $f = 0.02$, $\mu = 0.4$, and $\sigma = 3$, we derive a monotonic relationship between market size and the degree of differentiation. An increase in market size leads to more differentiation, to more variation, and to a larger number of firms (see figure 3). Furthermore, since the price index declines, consumers benefit from larger market in our numerical exercise.

A sensitivity analysis did not come up with different results. For a wide range of feasible numeric parameters, larger markets are served by more firms, with more varieties, and with more differentiation.

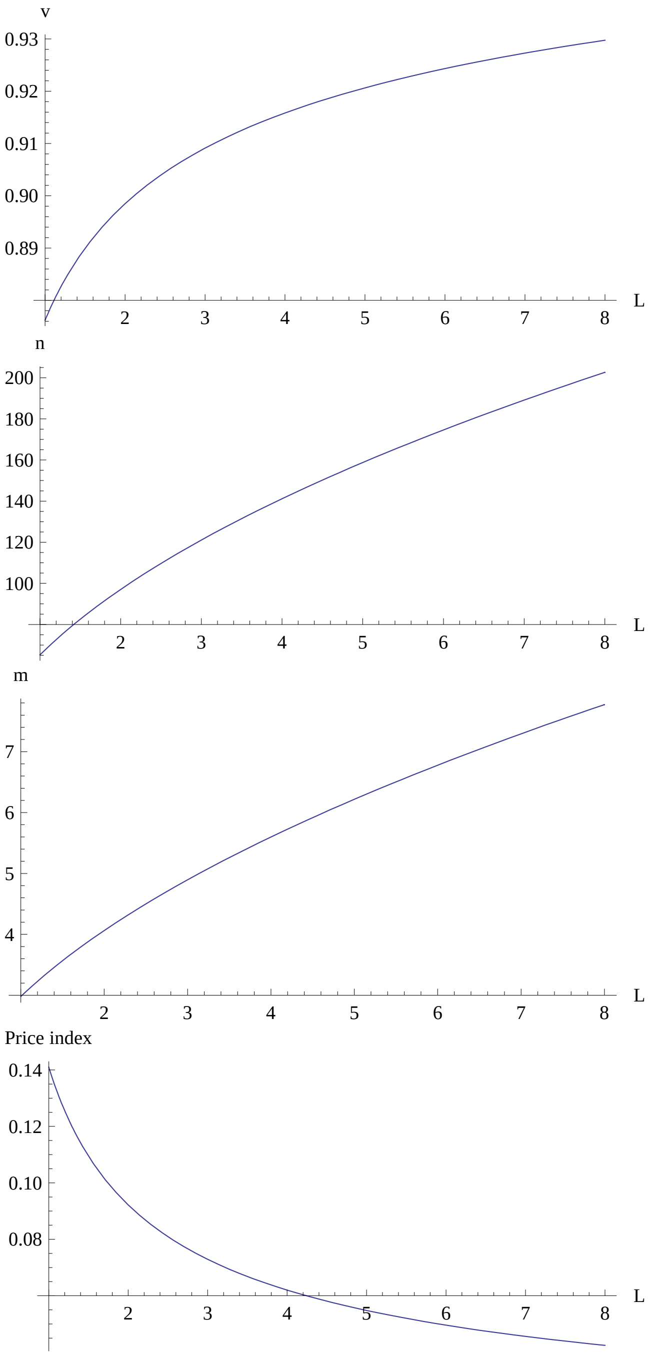

Figure 3: Market size, market equilibrium, and welfare

5 Concluding Remarks

Because of its analytical tractability and the strong results it delivers, the Dixit-Stiglitz model has become a workhorse for modern trade theory. This paper proposes an approach to incorporate horizontal product differentiation into this framework. We formulate a setting in which multi-product firms not only decide about entry and about the number of varieties they supply but also about the degree of differentiation between these varieties. This approach provides us with a much richer picture of firm behavior than the common model with just one product per firm. In this concluding section we do not review our results in detail. Instead, we emphasize what we regard as the central message from our paper: To evaluate the influence of trade on product variety it is not enough to look at the mere number of products. Instead, trade may also change the degree of differentiation between products and thereby affects variety. The next step toward a better understanding the variety effects of trade would be an empirical test of the effects of trade on the degree of product differentiation. We leave this step for future research.

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