International trade in a dynamic game model of renewable resource oligopoly

Kenji Fujiwara*
School of Economics, Kwansei Gakuin University

January 22, 2008

Abstract

Constructing a differential game model of international duopoly with renewable resources, this paper studies the effects of free trade. The linear and nonlinear feedback Nash equilibria are compared with the autarkic equilibrium. In contrast to the traditional view, we show that the opening of trade is anti-procompetitive, i.e., the world price exceeds the autarkic price. Furthermore, trade makes the resource stock lower as compared to the autarkic level. Due to these effects, each country loses from trade irrespective of strategies chosen. We discuss that these counter-intuitive results are highly intrinsic to the dynamic structure of our model.

Keywords: renewable resource, international oligopoly, feedback Nash equilibrium, gains from trade.

JEL classification: F10, F12, F18, Q27.

*School of Economics, Kwansei Gakuin University. Uegahara 1-1-155, Nishinomiya, Hyogo, 662-8501, Japan. Tel: +81-798-54-7066. Fax: +81-798-51-0944. E-mail: kenjifujiwara@kwansei.ac.jp.

1 Introduction

Why is there persistent resistance to globalization? As Bhagwati (2004) critically discusses, the harmful effect on the environment is one plausible motivation. It is quite popularly insisted that globalization and trade liberalization accelerate environmental degradation. However, this argument is not persuasive enough to support anti-globalization because there are still several sources of gains from trade despite the environmental degradation. In other words, we can assert that globalization is continues to be gainful if the positive effect of trade represented by classical welfare gains from trade overweighs the negative effect caused by environmental degradation.

The purpose of this paper is to provide a theoretical model to give an affirmative evaluation to the environmentalists' resistance to globalization. The model employed, which basically owes to Benchekroun (2003, 2007), is a dynamic game model of international duopoly with a transboundary renewable resource.[1] The model is a straightforward extension of Markusen's (1981) duopolistic model. There is one monopolistic firm in each country. When international trade begins, each monopolist plays a Cournot-Nash game in an integrated world market. The production of the good reduces the stock of renewable resource. The resource stock has an amenity value to the consumer as a public good. While Benchekroun (2003, 2007) focuses on the *linear* feedback (Markov perfect) Nash equilibrium, we look for the *nonlinear* feedback solutions as well as the linear feedback strategy.[2]

In this setting, we show two counter-intuitive effects of international trade. First, each firm reduces its output, thereby the steady state resource stock in free trade becomes smaller than that in autarky. This is quite curious because each firm reduces the output although the market structure changes from monopoly to duopoly. Due to the above effect, the international price turns out to be higher than the autarkic price, i.e., trade is anti-procompetitive, which is in sharp contrast to the well-known result in static models.[3] We should stress that this anti-procompetitive effect never occurs in a static model.

[1]Any interested reader in differential game theory is referred to Dockner *et al.* (2000).

[2]Within our framework, the open-loop solution can not be properly defined. Hence, our arguments will be based only on the feedback equilibria.

[3]See, for instance, Markusen (1981) and Helpman and Krugman (1985).

The second main result, which is established from the first result, concerns the welfare effect of trade. We show that free trade necessarily deteriorates the welfare as compared to the autarkic level in both the linear and nonlinear feedback Nash equilibria. While these losses-from-trade propositions hinge upon our specified model, they could be helpful to answer our basic question: why have environmentalists been resisting globalization?

The plan of this paper is as follows. Introducing a basic model, Section 2 describes an autarkic equilibrium. Section 3 derives and characterizes the linear and feedback Nash equilibria under free trade. Section 4 compares the equilibria and proves our main results summarized right above. Section 5 gives a concluding remark.

2 A model: autarkic equilibrium

The model we develop heavily owes to Benchekroun (2003). While he considers a closed economy, we extend his model to a two-country world. Consider a two-country (Home and Foreign), two-good (Goods 1 and 2), one-primary-factor (labor) model. An asterisk (*) is attached to the Foreign variables to distinguish them from the Home variables. Both goods are produced from labor. Without loss of generality, one unit of Good 2 is produced from one unit of labor. Then, as long as Good 2 is positively produced and its price is normalized to unity, the wage rate is also fixed to unity through the profit maximization condition. The production of Good 1 is monopolized under the constant returns technology. We assume that $cx, \ c > 0$ units of labor are employed to produce x units of Good 1. No fixed cost is incurred.

Concerning the demand side, assume a representative consumer whose preference is specified by

$$u = aC_1 - \frac{C_1^2}{2} + C_2 + bS, \quad a > c, \quad b > 0, \tag{1}$$

where u is the utility level, $C_i, \ i = 1, 2$ the consumption of each good, and S the stock of a renewable resource in the world. (1) tells us that the renewable resource has an amenity value as a positive externality to the consumer such as pure sea and clean air.[4]

[4]The term 'amenity value' is borrowed from Sorger (2005).

Maximizing (1) subject to the budget constraint, the demand function of Good 1 is

$$C_1 = a - p, \tag{2}$$

where p denotes the price of Good 1 measured by Good 2. Substituting (2) into (1) and rearranging, the Home welfare becomes

$$U = \frac{(a-p)^2}{2} + (p-c)x + bS, \tag{3}$$

where the first term in the right-hand side is the consumer surplus, and the second the monopolist's profit.

Based on the above specifications, let us solve the model of autarky. Under autarky, the market-clearing condition of Good 1 is

$$C_1 = a - p = x,$$

from which the Home monopolist's instantaneous profit is defined by

$$(p-c)x = (a-c-x)x.$$

Then, the Home monopolist's profit maximization problem can be formulated as

$$\begin{aligned} \max_x \quad & \int_0^\infty e^{-rt}(a-c-x)x dt, \quad r > 0 \\ \text{subject to} \quad & \dot{S} = kS - x - x^*, \quad k > 0, \end{aligned} \tag{4}$$

where r and k are the constant rate of discount and the natural growth rate of the resource, respectively. (4) gives the evolution of the world resource, according to which the resource declines as the aggregate production of Good 1 is positive and larger, whereas it exponentially grows with a natural rate of k.

Throughout this paper, we presume that the Home and Foreign monopolists are identical and behave symmetrically. We resort to the dynamic programming method to derive the optimality conditions. More specifically, we employ the technique for deriving the feedback (Markov perfect) strategy developed by Tsutsui and Mino (1990) and Shimomura (1991). It begins by defining the Hamilton-Jacobi-Bellmann equation:

$$rV(S) = \max_x \left\{(a-c-x)x + V'(S)[kS - x - x^*(S)]\right\}, \tag{5}$$

where $V(\cdot)$ is the Home firm's value function:

$$V(S) \equiv \max_x \left\{ \int_t^\infty e^{-r(\tau-t)}(a-c-x)xd\tau \;\Big|\; \dot{S} = kS - x - x^*(S) \right\}.$$

Note that the Foreign firm's output is a function of S because the Home firm expects that the rival chooses a feedback strategy, i.e., the state-dependent strategy in determining its own output.

Assuming an interior solution, maximization of the right-hand side in (5) implies that

$$V'(S) = a - c - 2x(S),$$

from the first-order condition. Substituting this into (5) and using the symmetry assumption such that $x = x^* = x(S)$, we have the following identity in S.

$$rV(S) = [a - c - x(S)]x(S) + [a - c - 2x(S)][kS - 2x(S)].$$

Differentiating both sides with respect to S and rearranging terms, we have reached the auxiliary differential equation:

$$x'(S) = \frac{(k-r)[2x(S) - (a-c)]}{6x(S) - 2kS - (a-c)}, \tag{6}$$

which gives the slope of the feedback strategy in the $S - x$ space. In the rest of this paper, we impose:[5]

Assumption. $k > 2r$.

From (6) and the steady state condition, we have a set of information:

$$\begin{aligned} x'(S) = 0 &\iff x(S) = \frac{a-c}{2} \\ x'(S) = \infty &\iff x(S) = \frac{2kS + a - c}{6} \\ \dot{S} = 0 &\iff x(S) = \frac{kS}{2}. \end{aligned}$$

These relationships are depicted as the loci of $x'(S) = 0,\ x'(S) = \infty$, and $\dot{S} = 0$ in Figure 1.

[5] A similar assumption is made in Dockner and Sorger (1996) and Benchekroun (2003) as well.

(Figure 1 around here)

Let us derive the equilibrium strategy in the present dynamic game. To begin with, the linear feedback strategy is obtained by the help of (6). Suppose that the strategy a linear function of S so that $x(S) = \alpha S + \beta$, where α and β are the undetermined coefficients. Then, $x'(S) = \alpha$ and (6) becomes

$$\alpha = \frac{(k-r)[2(\alpha S + \beta) - (a-c)]}{6(\alpha S + \beta) - 2kS - (a-c)},$$

or alternatively

$$\alpha(6\alpha - 4k + 2r)S + 2\beta[3\alpha - (k-r)] - [\alpha - (k-r)](a-c) = 0.$$

α and β need to be determined to satisfy this equation. Thus, the two pairs of (α, β) are obtained as follows.

$$(\alpha, \beta) = \left(0, \frac{a-c}{2}\right), \quad \left(\frac{2k-r}{3}, \frac{(-k+2r)(a-c)}{6k}\right).$$

In Figure 1, two linear strategies are depicted, one of which is given by x_1^L and the other of which x_2^L. Noting that S increases (resp. decreases) below (resp. above) the $\dot{S} = 0$ line, only x_1^L is asymptotically stable. That is, the static monopoly output, $x_2^L = (a-c)/2$, could be eliminated from the stability reasoning. Making use of the above two pairs of (α, β), the equilibrium strategy under autarky is explicitly derived as follows.

$$x(S) = \begin{cases} 0 & \text{if} \quad S \leq \frac{(k-2r)(a-c)}{2k(2k-r)} \\ \frac{2k-r}{3}S - \frac{(k-2r)(a-c)}{6k} & \text{if} \quad \frac{(k-2r)(a-c)}{2k(2k-r)} < S < \frac{(k-r)(a-c)}{k(2k-r)} \\ \frac{a-c}{2} & \text{if} \quad S \geq \frac{(k-r)(a-c)}{k(2k-r)}. \end{cases}$$

While there are multiple nonlinear strategies which satisfy (6) and the boundary condition, only the linear strategy x_1^L converges to the steady state. Accordingly, the unique steady state stock of resource is explicitly obtained as

$$S^A = \frac{a-c}{k}, \tag{7}$$

where the superscript A stands for the autarkic equilibrium.

We would like to comment that the nonlinear feedback strategy a la Tsutsui and Mino (1990) can not be properly defined in the present setting of autarky. That is, only the linear strategy can be obtained. However, extending this game to a two-country world, we can properly define and characterize nonlinear strategies. This is the main task in the next section.

3 Free trade equilibria

This section extends the above model to a two-country world. While the market structure exhibits monopoly under autarky, it changes to duopoly under free trade. Specifically, we assume that the two monopolists play a Cournot-Nash game in the integrated world market. We derive and characterize the feedback strategies arising in the free trade equilibria.

In the integrated market under free trade, the market-clearing condition is

$$C_1 + C_1^* = 2(a - p) = x + x^*,$$

from which the inverse demand function becomes

$$p = a - \frac{x + x^*}{2}.$$

Thus, the Home firm's problem is reformulated as

$$\begin{aligned} \max_x \quad & \int_0^\infty e^{-rt} \left(a - c - \frac{x + x^*}{2} \right) x dt \\ \text{subject to} \quad & \dot{S} = kS - x - x^*. \end{aligned}$$

While Benchekroun (2003) derives the linear feedback strategy in the closed economy version of this game, we allow for nonlinear feedback strategies. As in the previous section, the Tsutsui-Mino-Shimomura technique is adopted to this end. The Home firm's Hamilton-Jacobi-Bellmann equation is now replaced by

$$rV(S) = \max_x \left\{ \left[a - c - \frac{x + x^*(S)}{2} \right] x + V'(S)[kS - x - x^*(S)] \right\}. \tag{8}$$

Maximizing the right-hand side yields

$$V'(S) = a - c - \frac{3}{2} x(S).$$

Substituting this expression into (8) and making another use of the symmetry assumption, the following identity in S is obtained.

$$rV(S) = [a - c - x(S)]x(S) + \left[a - c - \frac{3}{2}x(S)\right][kS - 2x(S)].$$

Differentiating both sides with respect to S and rearranging terms, the slope of feedback strategies is given by the auxiliary differential equation:

$$x'(S) = \frac{(k-r)[3x(S) - 2(a-c)]}{8x(S) - 3kS - 2(a-c)}. \tag{9}$$

(9) identifies the following set of helpful information.

$$\begin{aligned} x'(S) = 0 \quad &\Longleftrightarrow \quad x(S) = \frac{2(a-c)}{3} \\ x'(S) = \infty \quad &\Longleftrightarrow \quad x(S) = \frac{3ks + 2(a-c)}{8} \\ \dot{S} = 0 \quad &\Longleftrightarrow \quad x(S) = \frac{kS}{2}. \end{aligned}$$

What makes the present case differ from that under autarky is that (asymptotically stable) nonlinear strategies can be properly defined as well as linear strategies. Figure 2 diagrammatically makes clear this point. As in Figure 1, there are three auxiliary lines: $x'(S) = 0$, $x'(S) = \infty$ and $\dot{S} = 0$. And there are two linear feedback strategies, one of which degenerates to the $x'(S) = 0$ locus and the other of which is given by the dashed line, x_1^L.

(Figure 2 around here)

Moreover, we can confirm that there are an uncountable number of nonlinear feedback strategies. Figure 2 depicts a few of them by integral curves. As we show below, we will pay special attention to only one of them represented by x^N since the steady state stock of resource associated with it is not only analytically solvable but it has an interesting property as has been pointed out by Tsutsui and Mino (1990) and Dockner and Long (1993).

In the rest of this section, we derive the linear and nonlinear feedback strategies with the aid of (9) and Figure 2.

3.1 Linear feedback strategies

Let us guess that the strategy is a linear function of S like $x(S) = \alpha S + \beta$. Then, we see that $x'(S) = \alpha$ and (9) becomes

$$\alpha = \frac{(k-r)[3(\alpha S + \beta) - 2(a-c)]}{8(\alpha S + \beta) - 3kS - 2(a-c)},$$

which leads to an alternative expression:

$$\alpha[8\alpha - 3(2k-r)]S + \beta[8\alpha - 3(k-r)] - 2[\alpha - (k-r)](a-c) = 0.$$

The two coefficients must be determined to satisfy this equation. Hence, we immediately have two pairs of (α, β) as follows.

$$(\alpha, \beta) = \left(0, \frac{2(a-c)}{3}\right), \quad \left(\frac{3(2k-r)}{8}, \frac{(-2k+5r)(a-c)}{12k}\right). \tag{10}$$

Substituting back each pair of (α, β) into $x(S) = \alpha S + \beta$ yields the linear feedback strategies. It is obvious that the former pair in (10) corresponds to a static Cournot-Nash equilibrium output, which is geometrically given by the horizontal line $x'(S) = 0$ in Figure 2. On the other hand, the strategy associated with the latter pair of (α, β) in (10) corresponds to the upward-sloping strategy, x_1^L, in the figure.

Summarizing the results obtained up to now, the linear strategy is explicitly derived as follows.[6]

$$x(S) = \begin{cases} 0 & \text{if} \quad S \leq \frac{2(2k-5r)(a-c)}{9(2k-r)} \\ \frac{3(2k-r)}{8} S + \frac{(-2k+5r)(a-c)}{12k} & \text{if} \quad \frac{2(2k-5r)(a-c)}{9(2k-r)} < S < \frac{10(a-c)}{9} \\ \frac{2(a-c)}{3} & \text{if} \quad S \geq \frac{10(a-c)}{9} \end{cases}$$

As is clear from Figure 2, there are two steady states under the linear strategies such as L and M. Point L (resp. M) is asymptotically stable (resp. unstable). Following the existing literature, it is fair to focus on the stable steady state L which is reached by the latter pair of (α, β) in (10). The steady state stock of resource is then computed by $\dot{S} = kS - 2(\alpha S + \beta) = 0$:

$$S^L = \frac{2\beta}{k - 2\alpha} = \frac{2(2k-5r)(a-c)}{3k(2k-3r)}, \tag{11}$$

where the superscript L represents the linear feedback Nash equilibrium achieved by x_1^L.

[6] This result heavily owes to Benchekroun (2003).

3.2 Nonlinear feedback strategies

Having derived the level of resource stock in the linear feedback Nash equilibrium, let us move on to the nonlinear feedback strategy. As already mentioned, there are an uncountable number of nonlinear feedback strategies which converge to the steady state. Among others, we stick to only one of them represented by x^N since the resource stock associated with it can be explicitly solved and easy to characterize.[7]

The steady state reached by x^N is given by N in Figure 2 on which locus of x^N is tangent to the $\dot{S} = 0$ line. Since the slope of the $\dot{S} = 0$ line is $k/2$ and the steady state output is $kS/2$, evaluating (9) at $x = kS/2$ and equating it to $k/2$, we have

$$\frac{k}{2} = \frac{(k-r)\left[\frac{3k}{2}S - 2(a-c)\right]}{kS - 2(a-c)}.$$

The left-hand side gives the slope of the $\dot{S} = 0$ line and the right-hand side the slope of x^N evaluated at $x = kS/2$. Solving this equation for S, the steady state stock at N proves to be

$$S^N = \frac{2(k-2r)(a-c)}{k(2k-3r)}, \tag{12}$$

where the superscript N denotes the nonlinear feedback Nash equilibrium. Note here that (12) is ensured to be positive under Assumption imposed in Section 2.

4 Comparison of equilibria

Based on the equilibrium outcomes derived in the previous sections, this section compares the autarkic equilibrium with the free trade equilibria. The following result will play a useful role in the subsequent arguments.

Proposition 1. *Comparing the steady state stock of renewable resource in the autarkic equilibria with that in the free trade equilibrium, the following ranking follows.*

$$S^A > S^N > S^L. \tag{13}$$

[7]Indeed, most of the existing literature such as Tsutsui and Mino (1990) and Dockner and Long (1993) focuses on x^N in their arguments.

Proof. Taking the ratio among S^A, S^L and S^N, respectively, we easily see that

$$\begin{aligned} \frac{S^L}{S^A} &= \frac{2(2k-5r)}{3(2k-3r)} < 1 \\ \frac{S^N}{S^A} &= \frac{2(k-2r)}{2k-3r} < 1 \\ \frac{S^N}{S^L} &= \frac{3(k-2r)}{2k-5r} > 1, \end{aligned}$$

for any k and r which satisfy Assumption made in Section 2. Hence, (13) is established. **Q. E. D.**

Making use of Proposition 1, one has no difficulty in examining the effect of free trade on the price and welfare. Let us first investigate whether free trade has a procompetitive effect, which is first found by Markusen (1981) in a static model without resource accumulation. It is stated in:

Proposition 2. *Free trade has an anti-procompetitive effect in the sense that it raises the equilibrium price relative to the autarkic price.*

Proof. The autarkic price, which is denoted by p^A, is obtained as

$$\begin{aligned} p^A &= a - x^A \\ &= a - \frac{kS^A}{2}, \end{aligned}$$

where the second line follows from the steady state condition with symmetry such that $\dot{S} = kS - 2x = 0$. In a similar manner, the free trade price denoted by p^T is

$$\begin{aligned} p^T &= a - \frac{x^T + x^{*T}}{2} \\ &= a - x^T \\ &= a - \frac{kS^T}{2}, \end{aligned}$$

where the second equation uses the symmetry assumption, i.e., $x^T = x^{*T}$, and the third the steady state condition. Thus, the dependence of the equilibrium price on the resource stock is the same under autarky and free trade. As a result, Proposition 1 yields the following ranking on prices:

$$p^L > p^N > p^A,$$

from which we can confirm the proposition. **Q. E. D.**

The welfare effect is straightforwardly obtained only by comparing the resource stock levels. This is established in:

Proposition 3. *Free trade is harmful to each country.*

Proof. In the steady state and from (3), the instantaneous utility becomes

$$\begin{aligned} U &= \frac{(a-p)^2}{2} + (p-c)x + bS \\ &= \frac{x^2}{2} + (a-c-x)x + bS \\ &= \frac{1}{2}\left(\frac{kS}{2}\right)^2 + \left(a-c-\frac{kS}{2}\right)\frac{kS}{2} + bS \\ &= \frac{S}{8}\left\{k[4(a-c)-kS]+8b\right\} \\ &\equiv U(S). \end{aligned}$$

As long as attention is confined to the symmetric equilibrium where $x = x^*$, the autarkic and free trade welfare levels depend on S identically, namely, both of them are measured by the same function $U(\cdot)$.

The function $U(\cdot)$ is strictly concave and its locus is mountain-shaped as depicted by Figure 3. Through the condition of $U'(S) = 0$, the locus reaches the peak at

$$S = \frac{2[k(a-c)+2b]}{k^2}.$$

Comparing this welfare-maximizing level of S with S^A in (7), it follows that

$$\frac{S}{S^A} = \frac{2[k(a-c)+2b]}{k(a-c)} > 1,$$

namely S^A is smaller than the resource stock at which the mountain reaches the peak. This, together with Proposition 1, implies that the relationship between the welfare and resource stocks is captured by Figure 3. From the figure, it is obvious that the welfare under autarky exceeds that under free trade. **Q. E. D.**

As Tsutsui and Mino (1990) and Dockner and Long (1993) stress, the nonlinear feedback Nash equilibrium reached at N in Figure 2 can mimic the collusive solution as $r \to 0$. Applying this finding to our context, the steady state N approximates the autarkic equilibrium when the discount rate is sufficiently close to zero. Hence, the welfare is also unchanged between S^A and S^N. However, once this 'limiting' assumption is ruled out, the welfare losses are inevitable.

5 Concluding remarks

This paper has presented a simple dynamic model of international duopoly with resource extraction. The linear and nonlinear feedback Nash equilibria are compared with the autarkic equilibrium. We have proved that international trade raises the world price relative to the autarkic price, namely, it is anti-procompetitive in contrast to the traditional view such as Markusen (1981).

Due to the above effect, international trade is welfare-deteriorating although the market structure changes from monopoly to duopoly. While these counter-intuitive results are based on a number of simplifying assumptions and settings, they could provide a theoretical rationale for why globalization has been resisted by environmentalists so persistently.

There are a variety of possible extensions of our model. Among others, our model can be extended to accommodate the model in which the resource stock affects not only the consumer's utility but the firm profit.[8] Within such a modified model, it might be possible to show the gains from trade, i.e., globalization is still beneficial.

[8]Note that each firm's profit is assumed not to be affected by the resource stock.

References

[1] Bhagwati, J. N. (2004), *In defense of globalization*, Oxford: Oxford University Press.

[2] Benchekroun, H (2003), 'Unilateral production restrictions in a dynamic duopoly,' *Journal of Economic Theory*, 111, 214-239.

[3] Benchekroun, H. (2007), 'Comparative dynamics in a productive asset oligopoly,' *Journal of Economic Theory*, forthcoming.

[4] Dockner, E. J. and N. V. Long (1993), 'International pollution control: cooperative versus noncooperative strategies,' *Journal of Environmental Economics and Management*, 25, 13-29.

[5] Dockner, E. J. and G. Sorger (1996), 'Existence and properties of equilibria for a dynamic game on productive assets,' *Journal of Economic Theory* 71, 209-227.

[6] Dockner, E., S. Jorgensen, N. V. Long and G. Sorger (2000), *Differential Games in Economics and Management Science*, Cambridge: Cambridge University Press

[7] Helpman, E. and P. R. Krugman (1985), *Market structure and foreign trade*, Cambridge, MA: MIT Press.

[8] Markusen, J. R. (1981), 'Trade and the gains from trade with imperfect competition,' *Journal of International Economics*, 11, 531-551.

[9] Shimomura, K. (1991), 'The feedback equilibria of a differential game of capitalism,' *Journal of Economic Dynamics and Control*, 15, 317-338.

[10] Sorger, G. (2005), 'A dynamic common property resource problem with amenity value and extraction costs,' *International Journal of Economic Theory*, 1, 3-19.

[11] Tsutsui, S. and K. Mino (1990), 'Nonlinear strategies in dynamic duopolistic competition with sticky prices,' *Journal of Economic Theory*, 52, 136-161.

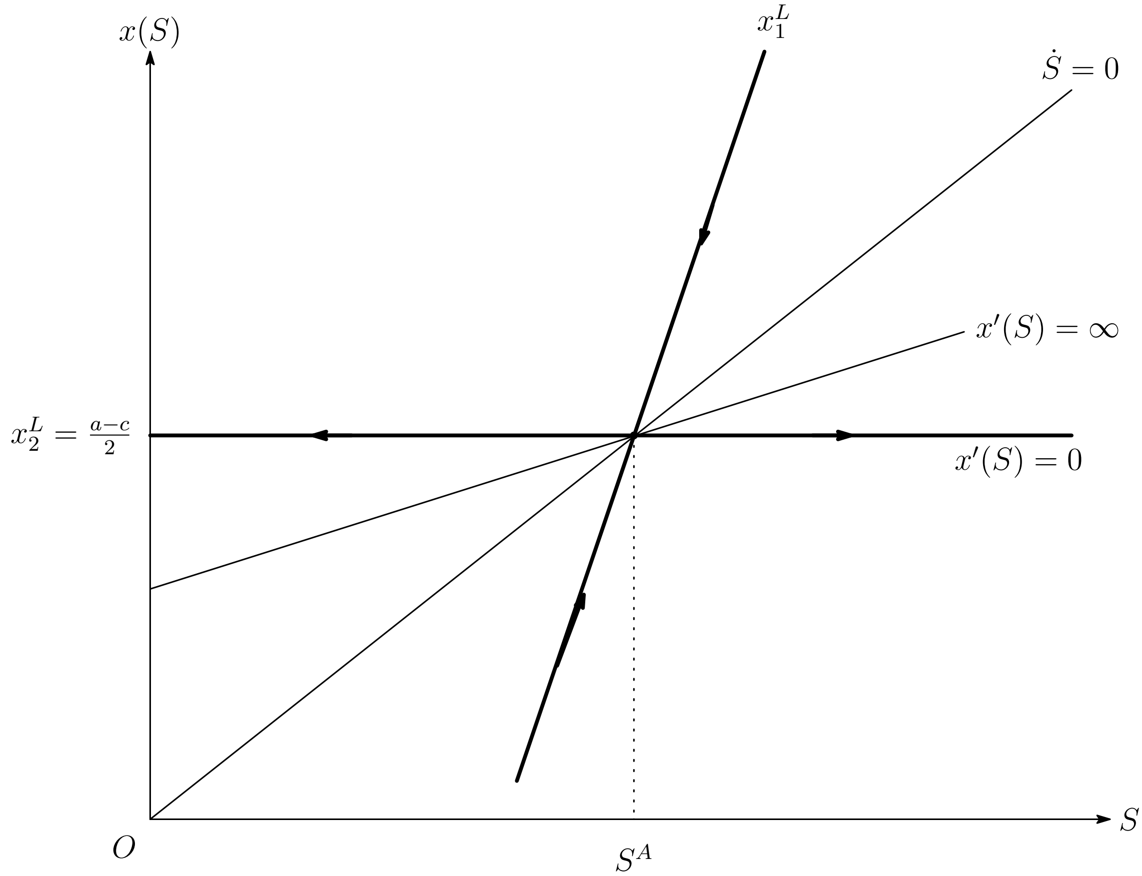

Figure 1: Equilibrium strategy under autarky

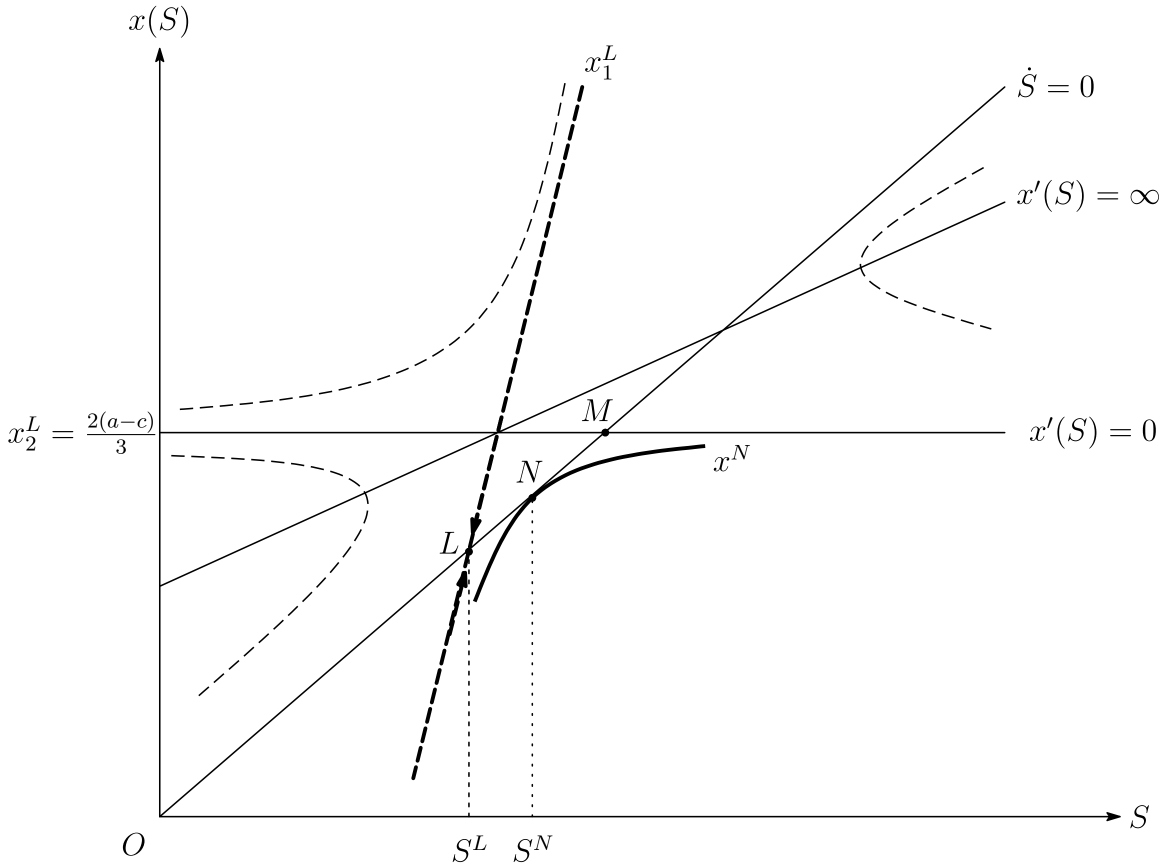

Figure 2: Equilibrium strategies under free trade

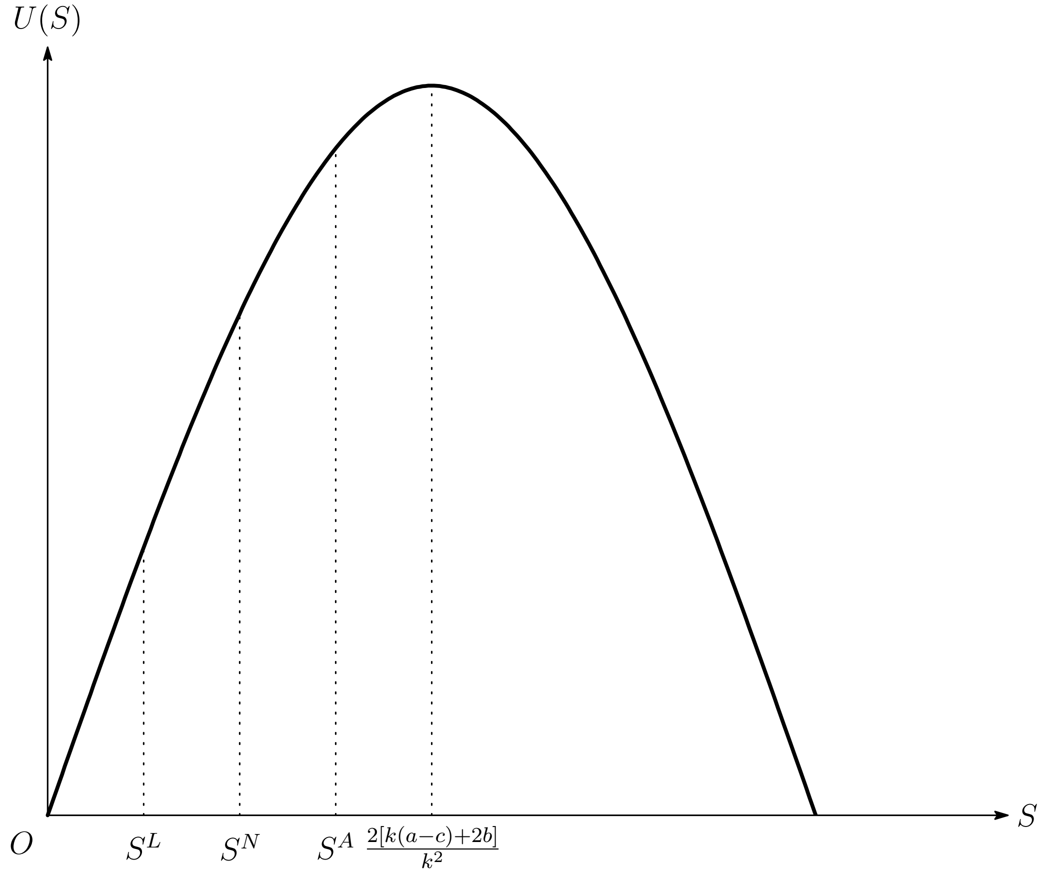

Figure 3: Resource stocks and welfare