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Directed Search without Price Directions

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Abstract

We present a simple directed search model of the labour market in which workers' outside options play a key role. Two versions of the model are considered: one exact, with finite numbers of workers, and one in the limit where the number of workers approaches infinity. The second version is an approximation of the first. We examine the effects of a set of government policy parameters and find that most of the influence of these parameters occurs through the channel of workers' outside options. This channel is fundamental in this model, and absent from others in the literature.

Key words: Directed search, matching, unemployment theory, public policy

JEL Codes: E24, J31, J41, J64, H20, D44

INTRODUCTION

In recent years, a new class of search-theoretic models of the labour market has appeared in which agents in the model are able to observe the location and characteristics of other agents, and *choose* who to approach, rather than simply accept arbitrary assignment by a matching technology. Due to the fact that agents in these models are *directed* by the characteristics they observe, this class of models has come to be known as "directed search" theory.[1] At the heart of most of these models is a simple coordination problem which is used to explain the co-existence of vacancies and unemployment, and the apparent constant returns to scale in estimated matching functions, without imposing these as *a priori* restrictions. Unemployment may exist even if there are equal numbers of identical buyers and sellers in the labour market, because it is possible, and indeed likely, that two buyers may inadvertently approach the same seller and consequently leave another seller without a buyer. According to this theory, this friction is enough to generate significant unemployment and vacancy rates, even when workers and firms are fully aware of each others' locations and prices. Moreover, in this type of setting, given an appropriate pricing mechanism and large enough markets, the equilibrium allocation is constrained-efficient in the sense that a planner could not improve on the allocation unless the planner was somehow able to reduce the coordination friction itself.

Different papers in this literature have modelled this coordination friction in the labour market somewhat differently. Montgomery (1991) and Burdett, Shi and Wright (2001), for example, modelled firms as sellers of jobs to workers who must choose which firms to approach. In the simplest version of that setting, each firm has one job to "sell", and each worker applies to only one job. Each firm posts a wage (the "price" of the job) and each worker chooses which firm to apply to, based on the observed wage postings. (Thus, worker applications in this setting are *directed* by price postings.) Workers are uncoordinated in their decisions about which firms to apply to. This lack of coordination leads to a focus on a mixed strategy equilibrium, where all workers randomize. The randomization generates the stochastic matching process.

[1] For surveys that cover directed search see King (2003) and Rogerson, Shimer and Wright (2004).

In Julien, Kennes, and King (2000a), alternatively, we cast workers in the role of sellers – sellers of labour, where *firms* must choose which workers to approach. In that setting, each worker has one unit of labour to sell and each firm can offer its vacancy to only one worker at a time. Each worker sells his labour through an auction mechanism, posting a reserve wage, and each firm chooses which worker to make its offer to, based on the observed reserve wage postings. (Thus, firms' offers in this setting are directed by reserve wage postings.) Firms are uncoordinated in their decisions about which workers to make offers to. This lack of coordination leads to a focus on a mixed strategy equilibrium, where all firms randomize. The randomization generates the stochastic matching process. In both of these settings, buyers are directed by some sort of price announcement by sellers, and buyers' mixed strategies are responsible for the imperfect assignment of buyers to sellers: some buyers and some sellers are expected to be unable to match – leading to some unfilled vacancies and unemployed workers.

Beyond the obvious formal similarities in these settings, there is an intriguing equivalence in the limit as the scale of the markets increases. In particular, as noted by Kultti (1999), expected payoffs to sellers are identical in this limit, whether they post prices or conduct auctions and post reserve prices.[2] The large (limit) models are also considerably easier to work with than their finite counterparts and it is only in this limit that efficient entry of vacancies is obtained (whether firms are sellers of jobs or buyers of labour).[3] The limit models, therefore, are potentially useful approximations for the finite-sized economies that we observe. They are also of particular interest because they fall under the class of economies analysed in McAfee (1993). He showed that, in this class of economies, where sellers can advertise competing direct mechanisms, the equilibrium involves sellers offering second price auctions with reserve prices set to their outside options. Moreover, in this setting, with quasi-linear preferences, any auction can be equivalently implemented using a posted price mechanism.[4]

[2] This is not true outside this limit, however: in Julien, Kennes, and King (2001) we show that for any finite numbers of buyers and sellers, sellers' expected payoffs are higher if all sellers auction.
[3] See Julien, Kennes, and King (2000b).
[4] We would like to thank to an anonymous referee for pointing this out.

Although these two models bear a striking similarity, especially in the limit, their pricing structure remains quite different. As pointed out by Albrecht, Gautier, and Vroman (2005), the reasons underlying efficient entry in the two models are almost polar opposites: perfect competition and perfect monopoly pricing. This is reflected in the fact that, in this limit, the equilibrium posted price reflects only labour market tightness, with no influence from outside options; whereas the equilibrium reserve price is driven precisely to the outside option.[5] In the limit economy with auctions, then, if outside options are known (as they typically are in these models), no reserve prices need to be announced. In this sense, in the auction model in the limit, search is directed not by prices but by the outside options themselves.

In this paper, we highlight this difference by examining the effects that government policy variables have in this environment. In particular, we focus on the effects that the tax and benefit system can have on workers' outside options and the consequent effects on vacancy entry and unemployment in an auction model. These effects are entirely absent in the wage posting model, since outside options play no role in the equilibrium of that model. We start by introducing a tax and benefit structure into the model with finite numbers of agents, and derive a set of two equations that determine the equilibrium values of the reserve wage and the number of vacancies that enter. We call this the "exact" economy, because it delivers exact results for any size of the economy. In this economy, the reserve wage is a function of candidates' outside options, but also reflects market tightness, productivity, and government policy parameters. The limit (or "approximate") economy is then derived and analysed. Here, the reserve wage is equal to candidates' outside options, which are affected by policy parameters. Finally, numerical simulations of the exact and approximate economies are presented, to gauge the accuracy of the approximation in the limit.

[5] See Burdett, Shi, and Wright (2001), equation (14) and Julien, Kennes, and King (2000a), equation (15).

We find that, in both the exact and approximate economies, for any given number of vacancies, the government policy parameters play no role in the matching process or the equilibrium unemployment rate. Thus, these parameters affect these variables only through their effect on the equilibrium number of vacancies. In the limit model, we also find, as one might expect, that employment subsidies increase the number of vacancies and reduce the number of unemployed. This effect exists in both the auction models and the wage posting models. We also find that unemployment benefits reduce vacancy creation, and increase unemployment. This effect, while sounding obvious, has a less obvious chain of causation: an increase in unemployment benefits increases candidates' outside options and hence the reserve wage. This increases the expected wage and, for any given number of vacancies, reduces firms' expected profit from vacancy creation. This reduces the number of vacancies and increases the unemployment rate. This effect would be completely absent in a wage-posting model. We also find that the income tax rate and the level of non-taxable income can both influence vacancy entry and unemployment rates. Again, both of these effects occur purely through the channel of their influence on the outside option, and would not appear in the wage posting environment. To complete the policy analysis, finally, we also identify optimal settings for the policy parameters, given that the government satisfies its budget constraint.

The remainder of the paper is structured as follows. Section 1 presents the exact, finite-agent economy. The limit approximation is presented in section 2. This is followed by the simulations in section 3 and the conclusion in section 4. The proof of the first proposition appears in the appendix.

1. THE EXACT ECONOMY

We consider a simple economy with a fixed large number N of identical, risk neutral, job candidates where each candidate has one indivisible unit of labor to sell. There are M vacancies, where $M \geq 0$, and is determined by free entry. The output from a worker is $y_0 = 0$ if unemployed and $y_1 = y > 0$ if employed. It costs an amount k to create a vacancy, where $0 < k < y$. Each vacancy can approach only one candidate.

The government in this economy controls four variables: the benefits paid to workers who are unemployed at the end of the period ($\theta \in [0, y]$), an employment subsidy paid to firms that hire workers ($\sigma \in [0, y]$), an income tax rate ($\tau \in [0,1]$), and a level of non-taxable income ($\omega \in [0, y]$). The government must also satisfy its budget constraint.

The order of play is as follows. First, the government sets the values of its policy parameters ($\theta, \sigma, \tau, \omega$). Then, given N job candidates, M vacancies enter the market. Once the number of entrants has been established, candidates choose their reserve wages r_n where $n \in \{1,2,...,N\}$ is used to index candidates. Observing the reserve wages, vacancies then choose which candidate to approach. Once vacancies have been assigned to candidates, wages are determined through the bidding game: candidates auction their labour services to the highest bidder. We solve the model using backwards induction.

1.1 *The Bidding Game*

Here, we take as given the policy parameters ($\theta, \sigma, \tau, \omega$), the number of vacancies M, the vector of reserve wages **r**, and the assignment of vacancies to candidates. Let $m_n \in \{1,2,...,M\}$ denote the number of vacancies bidding for candidate n. Let $w(r_n, m_n)$ denote the equilibrium wage obtained by candidate n. The (before-tax) outside option for each candidate is the benefit payment θ, and the outside option for the vacancy is zero.

For any given $r_n \in \mathbf{r}$, the ascending-bid auction generates the following after-tax wage distribution:

$$w(r_n, m_n) = \begin{cases} 0 & \text{if} \quad m_n = 0 \\ \min\{r_n(1-\tau) + \tau\omega, r_n\} & \text{if} \quad m_n = 1 \\ (y+\sigma)(1-\tau) + \tau\omega & \text{if} \quad m_n > 1 \end{cases} \tag{1.1}$$

Clearly, the *wage* for any candidate who is unemployed at the end of the period will be zero. If non-taxable income ω is no greater than the benefit payment θ, then the unemployed candidate's after-tax *payoff* will be $\theta(1-\tau) + \tau\omega$; if, however, $\omega > \theta$, then unemployed candidates receive θ after tax. Thus, the after-tax payoff for unemployed workers is: $\min\{\theta(1-\tau) + \tau\omega, \theta\}$.

If only one vacancy approaches a candidate then the worker receives his reserve wage r_n. If $\omega \leq r_n$ then his after-tax payoff is $r_n(1-\tau) + \tau\omega$; otherwise, the payoff is simply r_n. In general, then, the candidate's after-tax payoff is $\min\{r_n(1-\tau) + \tau\omega, r_n\}$.

If more than one vacancy approaches a candidate, then Bertrand competition drives the wage up to the point where the candidate receives all the output from the production, *y*, plus all of the subsidy, σ, that vacancies receive when they hire workers. Since $\omega \leq y$, then the after-tax payoff to candidates in this case is $(y+\sigma)(1-\tau) + \tau\omega$.

1.2 *Vacancies' Choice of Candidate to Approach*

Given the policy parameters ($\theta, \sigma, \tau, \omega$), the number of vacancies *M*, and the vector of reserve wages **r**, each vacancy chooses which candidate to approach. For any $r_n \in \mathbf{r}$ let

$\mathbf{r}_{-n}$ denote the vector of wages $(r_1, r_2, ..., r_{n-1}, r_{n+1}, ..., r_N)$. Let $p_n(r_n, \mathbf{r}_{-n})$ denote the probability that any particular vacancy approaches candidate *n*.[6] Thus, for each vacancy:

$$\sum_{n=1}^{N} p_n(r_n, \mathbf{r}_{-n}) = 1 \tag{1.2}$$

Also, given m_n identical offers, with symmetry, the probability that candidate *n* accepts any particular offer is $1/m_n$. By equation (1.1) above, if the vacancy is alone when it approaches candidate *n*, it must pay r_n, so its payoff will be $y + \sigma - r_n$. Alternatively, if the vacancy is not alone when approaching candidate *n,* then it must pay $y + \sigma$, so its payoff will be zero. Moreover, in any symmetric equilibrium, the probability that a vacancy will be alone when it approaches candidate *n* is $(1 - p_n(r_n, \mathbf{r}_{-n}))^{M-1}$. Thus, before knowing m_n, a vacancy's expected payoff from approaching candidate *n* is:

$$\Pi_n(r_n, \mathbf{r}_{-n}) = (1 - p_n(r_n, \mathbf{r}_{-n}))^{M-1}(y + \sigma - r_n) \tag{1.3}$$

In a mixed strategy equilibrium, each vacancy chooses $p_n(r_n, \mathbf{r}_{-n})$, *n* = 1, 2, …, *N*, so that $\Pi_n = \Pi$, $\forall n$. Let $p_{-n}(r_n, \mathbf{r}_{-n})$ denote the symmetric mixed strategy probability assigned to all other candidates, then equation (1.2) becomes:

$$p_{-n}(r_n, \mathbf{r}_{-n}) = \frac{1 - p_n(r_n, \mathbf{r}_{-n})}{N - 1}$$

Using this, together with equation (1.3), and the condition $\Pi_n = \Pi$, assuming that the reserve wage for all candidates other than *n* is equal to some value *r* (so $p_n(r_n, \mathbf{r}_{-n}) = p_n(r_n, r)$) we obtain:

[6] Since all vacancies are homogeneous and we focus on symmetric equilibria, to save on notation, we do not index their strategies.

$$p_n(r_n, r) = 1 - \frac{N-1}{1+(N-1)\left(\frac{y+\sigma-r_n}{y+\sigma-r}\right)^{\frac{1}{M-1}}} \quad (1.4)$$

Equation (1.4) presents the probability, for each vacancy, that it will visit candidate *n*, as a function of candidate *n*'s reserve wage r_n and the reserve wages for all other candidates, *r*. We now analyse this choice of reserve wages.

1.3 *Candidates' Reserve Wage Choices*

Given the policy parameters ($\theta, \sigma, \tau, \omega$) and the number of vacancies *M,* candidates choose their reserve wages to maximize their expected payoffs in a simultaneous move game with other candidates. Let q_{n0} and q_{n1} denote the probabilities that candidate *n* will receive zero offers and one offer respectively. Thus:

$$q_{n0}(r_n, r) = (1 - p_n(r_n, r))^M$$

and

$$q_{n1}(r_n, r) = Mp_n(r_n, r)(1 - p_n(r_n, r))^{M-1}$$

Where $p_n(r_n, r)$ is given in equation (1.4). The after-tax expected payoff function for candidate *n* is therefore given by:

$$V_n(r_n, r) = q_{n0}(r_n, r)\min\{\theta(1-\tau)+\tau\omega, \theta\} + q_{n1}(r_n, r)\min\{r_n(1-\tau)+\tau\omega, r_n\}$$
$$+ (1 - q_{n0}(r_n, r) - q_{n1}(r_n, r))((y+\sigma)(1-\tau)+\tau\omega) \quad (1.5)$$

Since candidates choose their reserve wages simultaneously, an equilibrium array of reserve wages is found by a standard Nash argument:

$$r_n^* = \arg\max_{r_n} V_n(r_n, r^*)$$

Proposition 1. The unique symmetric equilibrium reserve wage is:

$$r_n^* = r^* = \frac{(M-1)((y+\sigma)(1-\tau)+\tau\omega)+(N-1)^2 \min\{\theta(1-\tau)+\tau\omega,\theta\}}{(M-1)+(N-1)^2} \tag{1.6}$$

Proof: In the Appendix.

Equation (1.6) shows that the symmetric equilibrium reserve wage is a weighted average of the after-tax wage that the candidate would receive if more than one vacancy approached him $((y+\sigma)(1-\tau)+\tau\omega)$ and the candidate's outside option $(\min\{\theta(1-\tau)+\tau\omega,\theta\})$. Also, substituting $r_n^* = r^*$ into equation (1.4), we derive the result that, in the equilibrium, all vacancies assign equal probability to visiting each candidate:

$$p_n^*(r^*, r^*) = p^* = 1/N \tag{1.7}$$

We can now calculate the expected number of matches (x) in equilibrium: since $(1-p^*)^M$ is candidate i's probability of receiving no offers, then $1-(1-p^*)^M$ is the probability that candidate *i* will receive at least one offer. In the symmetric equilibrium, the expected number of matches is given by *N* times this number. Using the fact that $p^* = 1/N$:

$$x^*(N,M) = N \bullet \left(1-\left(\frac{N-1}{N}\right)^M\right) \tag{1.8}$$

The expected rate of equilibrium unemployment ($U^*(N,M)$) is simply the expected number of candidates who receive no offers, divided be the size of the labor force (here, the size of the labor force is the same as the number of candidates), and is given by:

$$U^*(N,M) = \left(\frac{N-1}{N}\right)^M \tag{1.9}$$

Notice that none of the policy parameters ($\theta, \sigma, \tau, \omega$) appear in equations (1.8) or (1.9). This leads to the next proposition.

Proposition 2: For any given number of vacancies *M*, in the symmetric equilibrium, the number of matches and the number of unemployed are independent of the government policy parameters.

Proof: Clear from inspection of equations (1.8) and (1.9).

The intuition for this result is quite straightforward. In the symmetric equilibrium, all vacancies announce the same reserve wage r^* (given in equation (1.6)), and so all vacancies assign the same probability of visiting to each candidate. Hence, matching occurs through the simple urn-ball process summarized in equations (1.8) and (1.9). As long as all candidates are identical in this way, this matching process will be obtained in equilibrium. As we show below, the policy parameters can affect vacancy profits, and hence affect the number of vacancies that enter; but, for any given number of entrants, these parameters will play no further role in matching or unemployment.

1.4 *The Entry of Vacancies*

Given the policy parameters ($\theta, \sigma, \tau, \omega$), firms choose how many vacancies to create. The substitution of equations (1.6) and (1.7) into equation (1.3) provides an expression for the expected payoff for a vacancy in the symmetric equilibrium, given that the vacancy has already been created:

$$\Pi^*(r^*) = \left(1 - \frac{1}{N}\right)^{M-1} \left(y + \sigma - r^*\right) \quad (1.10)$$

The cost of entry is given by the parameter *k*. The free entry condition is:

$$\Pi^*(r^*) - k = 0 \quad (1.11)$$

Together, equations (1.10) and (1.11) imply:

$$\left(\frac{N-1}{N}\right)^{M-1} \left(y + \sigma - r^*\right) = k \quad (1.12)$$

Equations (1.6) and (1.12) are two equations in two unknowns, M and r^*, that simultaneously determine the number of vacancies that enter and the reserve wage in the symmetric equilibrium. Given the set of policy parameters ($\theta, \sigma, \tau, \omega$), this completes the solution of the model.

1.5 *The Government's Budget Constraint*

To simplify the analysis, for the remainder of the paper, we impose the following restriction on the relative values of non-taxable income and unemployment benefits: $\omega \le \theta$.

This simplifies equations (1.1), (1.5), and (1.6) to:

$$w(r_n, m_n) = \begin{cases} 0 & if \quad m_n = 0 \\ r_n(1-\tau) + \tau\omega & if \quad m_n = 1 \\ (y+\sigma)(1-\tau) + \tau\omega & if \quad m_n > 1 \end{cases} \quad (1.1')$$

$$V_n(r_n, r) = q_{n0}(r_n, r)(\theta(1-\tau)+\tau\omega) + q_{n1}(r_n, r)(r_n(1-\tau)+\tau\omega)$$
$$+ \ (1 - q_{n0}(r_n, r) - q_{n1}(r_n, r))\big((y+\sigma)(1-\tau)+\tau\omega\big) \qquad (1.5')$$

$$r_n^* = r^* = \frac{(M-1)((y+\sigma)(1-\tau)+\tau\omega) + (N-1)^2(\theta(1-\tau)+\tau\omega)}{(M-1)+(N-1)^2} \qquad (1.6')$$

In the symmetric equilibrium, the government's budget constraint per candidate is:

$$\theta\left(\frac{N-1}{N}\right)^M + \sigma\left(1 - \left(\frac{N-1}{N}\right)^M\right) =$$
$$\tau\left((\theta-\omega)\left(\frac{N-1}{N}\right)^M + (r^*-\omega)\frac{1}{N}\left(\frac{N-1}{N}\right)^{M-1} + (y+\sigma-\omega)\left(1 - \left(\frac{N-1}{N}\right)^M - \frac{1}{N}\left(\frac{N-1}{N}\right)^{M-1}\right)\right)$$

(1.13)

Where r^* and M are determined by equations (1.6') and (1.12).

Analysing the comparative static effects of the government policy parameters $(\theta, \sigma, \tau, \omega)$ on the equilibrium values of the endogenous variables, although not impossible, is relatively difficult once the effects of entry are considered. To do this, one would totally differentiate equations (1.6') and (1.12) and solve for the relevant partial derivatives. At this point, the virtue of the approximation at the limit becomes quite apparent.

2. THE APPROXIMATE (LIMIT) ECONOMY

We now consider an economy that approximates the economy presented in Section 1, taken at the limit where the number of candidates is arbitrarily large. We consider three different cases. First, the economy with arbitrary values of the government policy parameters. Second, the economy with no government policy parameters. Thirdly, we consider the economy with government policy parameters set to their optimal values.

To analyse the limit economy, in any case, we start by defining labour market tightness as $\phi = M / N$.

2.1 *The Limit Economy with Arbitrary Government Policy Parameters*

The limit economy is considerably simpler to solve and analyse than the exact economy of Section 1 above. In particular, unlike the exact economy, the limit economy has simple closed-form solutions for the values of the key endogenous variables in the symmetric equilibrium.

Substitution of $M = \phi N$ into equations (1.6') and (1.7), for any ϕ, and taking the limit as $N \to \infty$, one obtains:[7]

$$r^* = \theta(1-\tau) + \tau\omega \qquad (2.6')$$

$$p^* = 0 \qquad (2.7)$$

Consider equation (2.7) first. This shows the straightforward result that, as the number of candidates gets large, given that the vacancies assign equal probability to approaching each candidate, the probability that any one employer will approach any particular candidate goes to zero.

[7] The derivation of (2.6') uses L'Hôpital's rule.

The reasoning behind equation (2.6') is less straightforward, but quite important. In this limit, workers' reserve wage announcements are driven down to their outside option ($\theta(1-\tau)+\tau\omega$). This result follows from the particular sequence of events assumed here (i.e., candidates move first in this game, by choosing their reserve wages) and the assumption that candidates can costlessly apply to all vacancies, while vacancies are restricted to making offers to only one candidate. With randomization, vacancies place less weight on approaching each candidate than each candidate places on approaching each vacancy. (Each candidate approaches each vacancy with certainty, but each vacancy approaches any candidate with some probability less than one.) Thus, as the market size increases, keeping the ratio of vacancies to candidates constant, there are asymmetric influences of increasing M and N : for each candidate, the probability of being approached erodes more quickly (as N increases) than the analogous probability for vacancies (as M increases). This then erodes the first-mover advantage that candidates enjoy when announcing their reserve wages – driving them down to the outside option.[8]

Using the fact that, for any $z \in \Re$, $\lim_{N\to\infty}(1+z/N)^N = e^z$, the equilibrium matching function and unemployment rate in the limit economy become, respectively:

$$x(N,\phi) = N(1-e^{-\phi}) \tag{2.8}$$

$$U^*(\phi) = e^{-\phi} \tag{2.9}$$

Similarly, the entry condition for vacancies (1.12) becomes:

$$e^{-\phi}(y+\sigma-r^*) = k \tag{2.12}$$

The government's budget constraint becomes:

[8] Equation (2.6') is also consistent with Theorem 2 in McAfee (1993). In the absence of any government policy parameters, the limit economy in this paper is a special case of the economy studied in McAfee's paper. (McAfee considers only a limit economy.)

$$(\theta(1-\tau)+\tau\omega)e^{-\phi}+\sigma(1-e^{-\phi})=\tau\big((r^*-\omega)\phi e^{-\phi}+(y+\sigma-\omega)(1-e^{-\phi}-\phi e^{\phi})\big) \quad (2.13)$$

Recall that, in the exact economy, r^* and M are determined simultaneously in equations (1.6) and (1.12). In the approximate economy equation (2.6') determines r^* independently of M. Substitution of (2.6') into (2.12) yields:

$$e^{-\phi}=\frac{k}{y+\sigma-\theta(1-\tau)-\tau\omega} \quad (2.14)$$

Equation (2.14) determines the equilibrium value of ϕ in this model. Taking the logarithm of both sides of this equation yields the symmetric equilibrium value of labour market tightness:

$$\phi^*=\ln\big(y+\sigma-\theta(1-\tau)-\tau\omega\big)-\ln k \quad (2.14')$$

Equilibrium vacancies $M^*=\phi^* N$ are therefore:

$$M^*=[\ln\big(y+\sigma-\theta(1-\tau)-\tau\omega\big)-\ln k]N \quad (2.15)$$

Also, substitution of (2.14) into (2.9) yields the symmetric equilibrium unemployment rate:

$$U^*=\frac{k}{y+\sigma-\theta(1-\tau)-\tau\omega} \quad (2.16)$$

Notice that, in general, both the number of vacancies and the unemployment rate are functions of all the government policy parameters $(\theta,\sigma,\tau,\omega)$. The following proposition summarizes the comparative static properties of this equilibrium.

Proposition 3: In the symmetric equilibrium of the limit economy:

a) Unemployment benefits θ decrease vacancies and increase unemployment.

b) Employment subsidies σ increase vacancies and decrease unemployment.

c) The non-taxable income level ω decreases vacancies and increases unemployment.

d) The effect of the income tax rate τ depends on the relative values of ω and θ. If $\omega < \theta$ then τ increases vacancies and decreases unemployment. If $\omega = \theta$ then income taxes have no effect on vacancies or unemployment.[9]

Proof: Follows from the evaluation of partial derivatives of equations (2.15) and (2.16). ■

To understand the effects that these policy variables have on unemployment, it is important to recall the results in Proposition 2: for any *given* number of vacancies *M*, the unemployment rate is *independent* of any of the policy parameters. Thus, the changes in the unemployment rate identified in Proposition 3 occur purely through the channel of altered vacancy creation.

Unemployment benefits affect the number of vacancies and the unemployment rate through their effect on candidates' outside options, and thereby the reserve wage r^* (from equation (2.6')). An increase in benefits raises the value of candidates' outside options (the reserve wage). Through the auction mechanism, this raises the wage received by candidates that have only one vacancy approach them and leaves unaffected the wage received by the candidates that have more than one vacancy approach them. This raises average wages, for any given number of vacancies, that entering vacancies can expect to pay. This lowers the expected payoff for each vacancy (the left hand side of equation (2.12)) and thus reduces the number of vacancies created in equilibrium. Through the matching process, this reduction in vacancies then increases the unemployment rate.

[9] Recall that, from Section 1.5 onwards, we have maintained the assumption that $\omega \leq \theta$.

Employment subsidies do not affect candidate's outside options or the reserve wage r^*, but do increase the payoff for vacancies that are alone when they approach a worker. (Vacancies that are not alone have the value of subsidy bid away by Bertrand competition, so the value of σ does not affect firm payoffs in this case.) An increase in σ will therefore increase the *ex ante* expected payoff for vacancies, for any given *M*, and thereby increase vacancy creation. Through the matching process, this increase in the number of vacancies decreases the unemployment rate.

The non-taxable income level ω influences candidates' outside options through its effect on *after-tax* benefit payments. An increase in ω will increase these after-tax payments and thereby increase the value of candidates' outside options. (For example, in the limit of the range considered here, where $\omega = \theta$, candidates receive the full value of the benefits.) The effects, then, are similar to the effect produced by increasing the level of benefits: the reserve wage (and, thereby, wages for candidates who are approached by only onevacancy) rises, this raises the average wages that vacancies can expect to pay, for any given number of vacancies. This lowers the expected payoff for each vacancy and thus reduces the number of vacancies created in equilibrium. Through the matching process, this reduction in vacancies then increases the unemployment rate

Changes in the income tax rate τ also influence vacancies and unemployment through the channel of changing candidates' outside options. This influence is felt only in the presence of benefit payments, and if and only if the non-taxable income level ω is less than the benefit payments θ. If $\omega = \theta$ then benefits are entirely untaxed, and changes in the tax rate have no effect on the value of candidates' outside option, which is simply θ in this case. If $\omega < \theta$ then an increase in the income tax rate will reduce candidates' after-tax outside option. This, then, will have the opposite effect of an increase in ω discussed in the previous paragraph: the reserve wage (and, thereby, wages for candidates who are approached by only one vacancy) falls, this reduces the average wages that vacancies can expect to pay, for any given number of vacancies. This increases the expected payoff for each vacancy and thus increases the number of vacancies created in equilibrium. Through the matching process, this increase in vacancies then decreases the unemployment rate.

2.2 *The Limit Economy with No Government*

It is instructive, at this point, to consider the special case when all the government policy parameters are set equal to zero. The key equations (2.6'), (2.12) and (2.16) become, respectively:

$$r^* = 0 \tag{2.6''}$$

$$e^{\phi^*} y = k \tag{2.12'}$$

$$U^* = \frac{k}{y} \tag{2.16'}$$

Equation (2.6") shows that, in this case, absent any other income, candidates' outside option is simply zero. Equation (2.12) shows the vacancy entry condition in this case. On the left side is a vacancy's *ex ante* expected revenue after wages have been paid. Due to the auction structure, a vacancy will receive positive revenue only if the vacancy is alone when approaching the candidate (as always, if more than one vacancy approaches the candidate, the revenue is bid away). The probability of this occurring is $e^{-\phi}$. When vacancies are alone in approaching a candidate, their revenues are y and they pay the worker the outside option, which is zero. Thus, the left hand side of (2.12') presents vacancies' expected revenues after wages have been paid. The right hand side is the cost of the vacancy. Recognizing that $e^{-\phi}$ is also the unemployment rate, equation (2.16') then presents a very natural condition for equilibrium unemployment: this is given by the ratio of the cost of a vacancy to the output produced by the vacancy when filled.

2.3 *Optimal Government Policy*

Consider, first, a planner that is somehow able to control the entry of vacancies, but still faces the same coordination friction as the private agents. With this direct control, the planner then chooses ϕ to maximize total expected surplus:

$$S = N\left((1-e^{-\phi})y - \phi k\right) \tag{2.17}$$

The first order condition from this maximization problem is:

$$e^{-\phi} y = k \tag{2.18}$$

Which is identical the condition (2.12') above.[10] This illustrates the (now well-known) result that, in the absence of any distortionary structures, the decentralized equilibrium of large directed search economies is constrained-efficient.[11] Thus, setting all the policy variables equal to zero is an optimal government policy in this framework.

It is worth noting, however, that this policy exposes candidates to the full income risk associated with the random matching process. While it is true that candidates are risk neutral in this economy, it is still natural to question whether or not an alternative policy setting may exist which preserves the constrained efficiency of the zero policy setting, but also eliminates the income risk. This question is answered in the following proposition.

[10] The concavity of the surplus function ensures that the first order condition identifies a global maximum.
[11] Notice that this is not true in the exact economy – only in the limit.

Proposition 4: In the symmetric equilibrium of the limit economy, the following policy settings maximize expected surplus, eliminate income risk for candidates, and satisfy the government's budget constraint:

$$\sigma = \omega = \theta = y\left(1 - e^{-\phi} - \phi e^{-\phi}\right) \quad \text{and} \quad \tau = 1 .$$

Proof: Setting $\sigma = \omega = \theta$ and $\tau = 1$ in equation (2.14) yields equation (2.18), and in equation (2.6') yields $r^* = \theta$. Using these in the government budget constraint (2.13) yields the after-tax payment to unemployed workers $\theta = y\left(1 - e^{-\phi} - \phi e^{-\phi}\right)$. From equation (1.1'), workers who receive a visit from one vacancy obtain wage $r^*(1-\tau) + \tau\omega$ which, using the above values becomes $\omega = \theta = y\left(1 - e^{-\phi} - \phi e^{-\phi}\right)$. Also from equation (1.1'), workers who receive visits from more than one vacancy obtain the wage $(y+\sigma)(1-\tau) + \tau\omega$ which, using the above values, becomes $\omega = \theta = y\left(1 - e^{-\phi} - \phi e^{-\phi}\right)$. ■

This policy configuration, for all its virtues, is quite drastic. The tax rate, above the critical income of ω, is set at 100%. The value of ω is set equal to the value of unemployment benefits. Thus, after-tax, all workers receive the same income as the unemployed. Since candidates' outside options are driven up to this same value, to re-establish efficient vacancy entry, the employment subsidy to firms must be equal to precisely the same amount. The government budget constraint then determines what that amount is. The starkness of this optimal configuration is due largely to the fact that all workers are homogeneous in this model so any income disparity reflects only the luck of the draw – whose influence insurance is designed to eliminate. A model where workers invest in skills at a prior stage would alter this result to some degree.[12]

[12] See Julien, Kennes, and King (2005) for one such model.

3. THE ACCURACY OF THE APPROXIMATION

To get a sense of how quickly the exact economy converges to its limit as the size of the economy increases, we simulated the model, for particular values of the parameters, and for a range of values of *N*. For simplicity, we set all of the policy parameters equal to zero. The two parameters left in the model are *y* and *k*. As ballpark figures, for weekly incomes, we set:

$$y = 500 \qquad\qquad k = 50.$$

Using equations (2.6"), (2.12') and (2.16'), respectively, from section 2.2 above, we find the following numbers for the symmetric equilibrium reserve wage, market tightness, and unemployment in the limit economy:

$$r^* = 0 \qquad \phi^* = 2.3026 \qquad U^* = 0.1$$

Figure 1 plots the values of r^* from the exact economy, for $N = 2,...,100$, using equation (1.6') and (1.12), with the policy parameters set at zero.

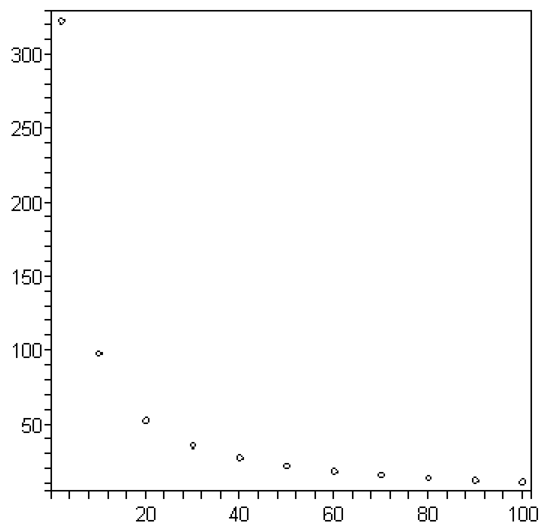

Figure 1: Reserve Wages in the Exact Economy

For small values of N, the reserve wage in the exact economy is quite high relative to the value of y. In the smallest case, where $N = 2$, $r^* = 322.95$, which is approximately 65% of output. This drops relatively quickly as N increases, but is still fairly significant, even for some relatively large values of N. For example, when $N = 20$: $r^* = 52.90$, or approximate 10.6% of y. When $N = 100$, then $r^* = 11.31$, or 2.3% of y. Even at $N = 200$ (not shown in Figure 1) r^* is not negligible: 5.71, or 1.14% of y.

Figure 2 plots the values of equilibrium market tightness ϕ^* from the exact economy, for $N = 2,...,100$, using equations (1.6') and (1.12), with the policy variables set at zero:

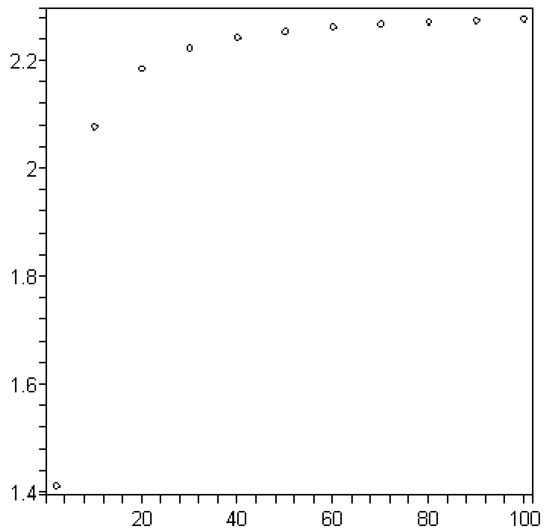

Figure 2: Market Tightness in the Exact Economy

When $N = 2$, $\phi^* = 1.14$, only 49% of the limit value of ϕ^*. Once N increases to 10, $\phi^* = 2.0781$, approximately 90% of the limit value. At $N = 50$, $\phi^* = 2.2545$, or 98% of the limit value. At $N = 100$, $\phi^* = 2.2783$, or 99%. At $N = 200$, $\phi^* = 2.2904$, or 99.5%. Thus, market tightness converges quite quickly.

Figure 3 plots the values of the equilibrium unemployment rate U^* from the exact economy, for $N = 2,...,100$, using equation (1.9), once M^* has been determined from (1.6') and (1.12):

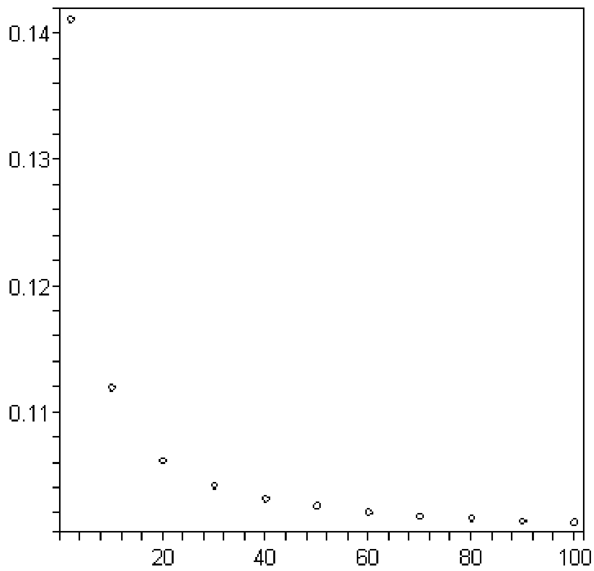

Figure 3: The Unemployment Rate in the Exact Economy

When $N = 2$, $U^* = 0.1412$, or 141.2% of the limit value of ϕ^*. Once N increases to 10, $U^* = 0.1120$, 112% of the limit value. At $N = 50$, $U^* = 0.1026$, or 102.6% of the limit value. At $N = 100$, $U^* = 0.1013$, or 101.3%. At $N = 200$, $U^* = 0.1006$, or 100.6%. Thus, unemployment converges quite quickly.

Overall, the limit approximation gives a reasonably accurate (within 1% of their true values) estimate of market tightness and unemployment in exact economy, for any $N > 100$. Similar accuracy of the reserve wage approximation requires a market of somewhat larger size: $N > 200$.

4. CONCLUSION

The directed search model examined here is very simple, particularly in the limit, but has quite rich policy implications. A key feature of this model, which makes it distinct from other models of this type, is the role of workers' outside options. In the exact model presented here, with finite numbers of market participants, these options influence the reserve wage that directs firms' offers. In the limit approximation, the outside option is the sole determinant of the reserve wage. In this sense, in this model, search is directed by these outside options rather than any strategic pricing decision by workers.[13]

We found that much of the influence that government policy parameters have in this model works through their influence on workers' outside options. In particular, the impacts of changes in unemployment benefits, income tax rates, and the level of non-taxable income work exclusively through this channel. In each case, these affect average expected wages, firm profits, vacancy entry and, finally, unemployment. We also identified a configuration of the policy parameters in this environment that induces constrained-efficient entry, eliminates income risk, and satisfies the government's budget constraint. The starkness of this policy points to ways in which this model could be fruitfully extended: including worker effort and skill accumulation, or heterogeneity on the other side of the market.

The appeal of the limit model, for all of its simplicity, must be tempered by the concern about how well it approximates the model with finite numbers of agents in it. Numerical simulations of the latter model reveal that, for the former to be a good approximation, the labour market in question should have at least hundreds of participants.

[13] Moreover, in a game with coordination frictions, the sellers' outside option is generally stochastic with many possible realizations (see Julien, Kennes and King (2006)). The auction model gives a clear and simple prediction about how these realizations influence pricing.

Appendix

Proof of Proposition 1:

All workers $n = 1,...,N$ simultaneously solve the following problem:

$$\max_{r_n} V_n(r_n, r^*)$$

The first-order conditions simplifies to

$$q_{n1}(r_n, r^*)\left(1 + \frac{\Phi}{p_n(r_n, r^*)(1 - p_n(r_n, r^*))}\frac{\partial p_n(r_n, r^*)}{\partial r_n}\right) = 0, \quad \forall n = 1,...,N$$

where $q_{n1}(r_n, r^*)$ is defined previously and

$$\Phi = (M-1)p_n(r_n, r^*)((y+\sigma)(1-\tau) + \tau\omega - r_n) + (1 - p_n(r_n, r^*))(r_n - \min\{\theta(1-\tau) + \tau\omega, \theta\}).$$

Substituting

$$\frac{\partial p_n(r_n, r^*)}{\partial r_n}$$

$$= -\frac{(N-1)((y+\sigma)(1-\tau) + \tau\omega - r_n)^{1/(M-1)-1}(1 - p_n(r_n, r^*))}{(M-1)\left[((y+\sigma)(1-\tau) + \tau\omega - r^*)^{1/(M-1)} + (N-1)((y+\sigma)(1-\tau) + \tau\omega - r_n)^{1/(M-1)}\right]}$$

into the first-order condition and exploiting symmetry setting $r_n = r^*$ and $p_n(r_n, r^*) = 1/N, \forall n$, yields equation (1.6).

Uniqueness is shown by demonstrating that $V_n(r_n, r^*)$ is strictly concave in r_n over $[\min\{\theta(1-\tau) + \tau\omega, \theta\}, (y+\sigma)(1-\tau) + \tau\omega]$. This demonstration is straightforward but cumbersome and the authors will gladly provide it upon request. ■

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